

Disentangling CP phases in nearly degenerate resonances: neutralino production via Higgs at a muon collider

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ABSTRACT: In the CP-violating Minimal Supersymmetric Standard Model, we study the pair production of neutralinos at center-of-mass energies around the heavy neutral Higgs boson resonances. For longitudinally polarized muon beams, we analyze CP asymmetries which are sensitive to the interference of the two heavy neutral Higgs bosons. Due to radiatively induced scalar-pseudoscalar transitions, the CP asymmetries can be strongly enhanced when the resonances are nearly degenerate, as in the Higgs decoupling limit. The Higgs couplings to the neutralino sector can then be analyzed in the presence of CP violating phases. We present a detailed numerical analysis of the cross sections, neutralino branching ratios, and the CP observables. We find that radiatively induced CP violation in the Higgs sector leads to sizable CP-asymmetries, which are accessible in future measurements at a muon collider. However, we expect that our proposed method should be applicable to other processes with nearly degenerate scalar resonances, even at hadron colliders.

KEYWORDS: Supersymmetry Phenomenology, Supersymmetric Standard Model, Higgs Physics.

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1. Introduction

The CP-conserving Minimal Supersymmetric Standard Model (MSSM) contains three neutral Higgs bosons [1–5], the lighter and heavier CP-even scalars h and H , respectively, and the CP-odd pseudoscalar A . While the MSSM Higgs sector is CP-conserving at Born level even in the presence of CP-phases, loop effects mediated dominantly by third generation squarks can generate significant CP-violating scalar-pseudoscalar transitions, leading to mixing of the neutral Higgs states into the mass eigenstates H_1, H_2, H_3 , with no definite CP parities [6, 8–14].

It is well known that mixing of states with equal conserved quantum numbers is strongly enhanced when these states are nearly degenerate, i.e., when their mass difference is of the order of their widths [9, 10]. This degeneracy occurs naturally in the Higgs decoupling limit of the MSSM, where the lightest Higgs boson has Standard Model-like couplings and decouples from the significantly heavier Higgs bosons [15]. In the decoupling limit, a resonance enhanced mixing of the states H and A can occur, which may result in nearly maximal CP-violating effects [8–10]. The general formalism for mass mixing in extended Higgs sectors with explicit CP violation is well developed [11–14], and sophisticated computer codes are available for numerical calculations [16–18]. Detailed investigations of the fundamental properties of the Higgs bosons, both phenomenological and experimental, will be crucial for the understanding of the mechanism of electroweak symmetry breaking.

In previous studies of the CP-conserving [19–27] and CP-violating Higgs sector [28–35], it was shown that the CP-properties and couplings of the heavy neutral Higgs bosons can be ideally tested in $\mu^+\mu^-$ collisions. Such a muon collider is a superb machine for measuring the neutral Higgs masses, widths, and couplings with high precision, since the Higgs bosons are resonantly produced in the s -channel [36–38]. The well controllable beam energy allows the study of the center-of-mass energy dependence of observables around the Higgs resonances. In particular, the beam polarization plays an essential role for analyzing the CP nature of the Higgs sector itself. Not only backgrounds can be reduced, but the CP-even and CP-odd contributions of the interfering Higgs resonances to the observables can be ideally studied, if the beam polarizations are properly adjusted [23–33].

Besides the initial beam polarization, the final fermion polarizations are essential to probe the Higgs interference. The secondary decays of the final fermions enable their spin analysis, and additional final state polarization observables allow for a complete determination of the CP-properties of the Higgs bosons [25–28]. For final state SM fermions $f\bar{f}$, with $f = \tau, b, t$, such polarization observables have been classified according to their CP and $C\tilde{T}$ ¹ transformation properties [28]. For the production of neutralinos [26] and charginos [27] with longitudinally polarized beams, it has been shown that asymmetries in the energy distributions of their decay products are sensitive to the Higgs interference of the CP-even and CP-odd production amplitudes. Thus the couplings of the Higgs boson to the neutralino and chargino sector can be analyzed in the CP-conserving MSSM.

¹ \tilde{T} is the naïve time reversal $t \rightarrow -t$, which inverts momenta and spins without exchanging initial and final particles.

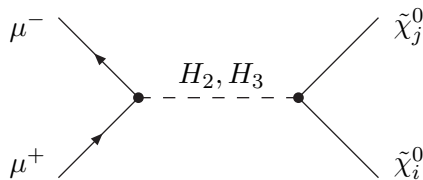


Figure 1: Resonant Higgs exchange in neutralino pair production.

In this work, we extend the study of neutralino production at the muon collider [26] to the CP-violating case. For neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0$, with longitudinally polarized muon beams, we define a CP-odd polarization asymmetry. We analyze the longitudinal polarizations of the produced neutralinos by their subsequent leptonic two-body decays $\tilde{\chi}_j^0 \rightarrow \ell\tilde{\ell}_n$, $\ell = e, \mu, \tau$, with $n = R, L$ for $\ell = e, \mu$, and $n = 1, 2$ for $\ell = \tau$. With the energy distribution of the leptons we can define a CP-even and a CP-odd polarization asymmetry for the neutralino decay, which probe the neutralino polarization. First results for neutralino and also chargino production in the MSSM with explicit CP violation in the Higgs sector have been reported in ref. [7].

In section 2, we give our formalism for neutralino production and decay with longitudinally polarized beams. In an effective Born-improved approach, we include the leading self energy corrections into the Higgs couplings. We give analytical formulas for the production and decay cross sections and distributions, and show that the energy distribution of the neutralino decay products depends sensitively on the Higgs interference. In section 3, we define CP-odd and CP-even asymmetries of the production cross section and of the energy distributions. These observables are sensitive to the CP-phases in the Higgs sector, as well as to absorptive contributions from the Higgs boson propagators. In section 4, we present a detailed numerical analysis of the cross sections, neutralino branching ratios and the CP-observables. We give special attention to the \sqrt{s} dependence of the observables and analyze their dependence on the CP violating phase ϕ_A , of the common trilinear scalar coupling parameter $A_t = A_b = A_\tau \equiv A$, and on the gaugino and higgsino mass parameters M_2 and μ , which mainly determine the Higgs couplings to the neutralinos. We summarize and conclude in section 5.

2. Neutralino production and decay formalism

We study CP violation in the Higgs sector in pair production of neutralinos

$$\mu^+ + \mu^- \rightarrow \tilde{\chi}_i^0 + \tilde{\chi}_j^0, \quad (2.1)$$

with longitudinally polarized muon beams. The Feynman diagram for Higgs boson exchange is shown in figure 1. We will analyze the process at center-of-mass energies of the nearly mass degenerate heavy neutral Higgs bosons H_2 and H_3 . They will be resonantly produced in the s -channel. The significantly lighter Higgs boson H_1 and the Z -boson are also exchanged in the s -channel, however far from their resonances. Together with the smuon exchange $\tilde{\mu}_{L,R}$ in the t - and u -channels, they contribute the non-resonant continuum to the neutralino production, see their Feynman diagrams in figure 2.

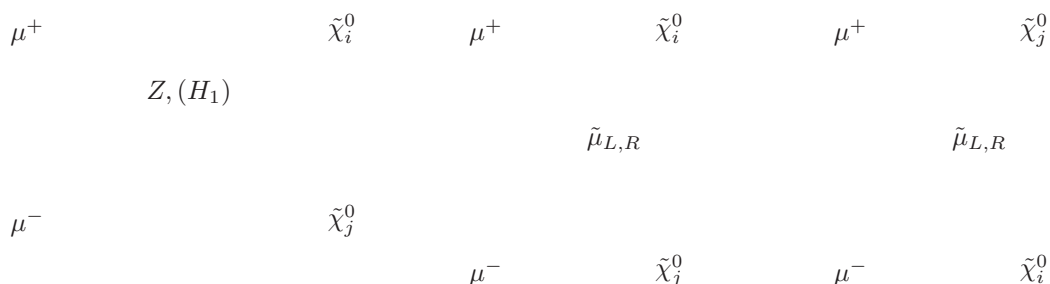


Figure 2: Feynman diagrams for non-resonant neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0$.

To analyze the longitudinal polarizations of the produced neutralinos, we consider their subsequent CP-conserving but P-violating leptonic two-body decays

$$\tilde{\chi}_j^0 \rightarrow \ell^\pm + \tilde{\ell}_n^\mp, \quad \ell = e, \mu, \tau, \quad (2.2)$$

with $n = R, L$ for $\ell = e$ or $\ell = \mu$, and $n = 1, 2$ for $\ell = \tau$.

2.1 Lagrangians and amplitudes for Higgs exchange

CP violation of the MSSM Higgs sector is induced by scalar-pseudoscalar mixing at the loop level. We will include these mixings effectively in the interaction Lagrangians for neutralino production (2.1) via Higgs exchange H_k , with $k = 1, 2, 3$,

$$\mathcal{L}_{\mu^+\mu^-H} = \bar{\mu} \left[c_L^{H_k\mu\mu} P_L + c_R^{H_k\mu\mu} P_R \right] \mu H_k, \quad (2.3)$$

$$\mathcal{L}_{\tilde{\chi}\tilde{\chi}H} = \frac{1}{2} \bar{\tilde{\chi}}_i \left[c_L^{H_k\chi_i\chi_j} P_L + c_R^{H_k\chi_i\chi_j} P_R \right] \tilde{\chi}_j H_k. \quad (2.4)$$

We obtain the effective Higgs couplings to the initial muons, $c_{L,R}^{H_k\mu\mu}$, and the final neutralinos, $c_{L,R}^{H_k\chi_i\chi_j}$, from their tree level couplings

$$c_{L,R}^{H_k\mu\mu} = C_{k\alpha} c_{L,R}^{h_\alpha\mu\mu}, \quad (2.5)$$

$$c_{L,R}^{H_k\chi_i\chi_j} = \tilde{C}_{k\alpha} c_{L,R}^{h_\alpha\chi_i\chi_j}, \quad h_\alpha = h, H, A, \quad (2.6)$$

with

$$\tilde{C} = C^{-1T}. \quad (2.7)$$

The tree level couplings $c_{L,R}^{h_\alpha\mu\mu}$ and $c_{L,R}^{h_\alpha\chi_i\chi_j}$ are defined and discussed, e.g., in refs. [26, 3]. The matrix C diagonalizes the Higgs mass matrix \mathbf{M} at fixed p^2 . In the tree-level basis of the CP-eigenstates h, H, A , the symmetric and complex mass matrix at momentum squared p^2 is given by [16]

$$\mathbf{M}(p^2) = \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}. \quad (2.8)$$

Here $\hat{\Sigma}_{rs}(p^2)$ with $r, s = h, H, A$ are the renormalized self energies of the Higgs bosons at one loop, supplemented with higher-order contributions, see ref. [16]. When the Higgs bosons are nearly mass degenerate, these corrections are enhanced by the Higgs mixing. The propagator matrix

$$\Delta_{rs}(p^2) = -i [p^2 - \mathbf{M}(p^2)]_{rs}^{-1}, \quad (2.9)$$

has complex poles at $p^2 = \mathcal{M}_{H_k}^2 \equiv M_{H_k}^2 - iM_{H_k}\Gamma_{H_k}$, $k = 1, 2, 3$, where M_{H_k} and Γ_{H_k} are the mass and width of the Higgs boson mass eigenstate H_k , respectively. We evaluate the mass matrix $\mathbf{M}(p^2)$ at fixed $p^2 = M_{H_2}^2$ in its Weisskopf-Wigner form with the program `FeynHiggs 2.5.1` [16, 17], in order to obtain the diagonalization matrix C , as well as the Higgs masses and widths. Here we neglect the momentum dependence of $\mathbf{M}(p^2)$, since, unless Higgs decay thresholds open at this energy, this dependence is weak in the resonance region $p^2 \approx M_{H_2}^2, M_{H_3}^2$. This approach corresponds to an improved-Born approximation. It includes the leading-order radiative corrections into the matrix C , but not the specific vertex and box corrections, as well as the subleading muon and neutralino self energy corrections. We give further details in appendix A.

With the Born-improved effective couplings we write the amplitudes for neutralino production via Higgs exchange, see figure 1,

$$\begin{aligned} T^P &= \Delta(H_k) \left[\bar{v}(p_{\mu^+}) \left(c_L^{H_k\mu\mu} P_L + c_R^{H_k\mu\mu} P_R \right) u(p_{\mu^-}) \right] \\ &\quad \times \left[\bar{u}(p_{\chi_j^0}) \left(c_L^{H_k\chi_i\chi_j} P_L + c_R^{H_k\chi_i\chi_j} P_R \right) v(p_{\chi_i^0}) \right], \end{aligned} \quad (2.10)$$

with the Breit-Wigner propagator for the Higgs boson

$$\Delta(H_k) = \frac{i}{s - M_{H_k}^2 + iM_{H_k}\Gamma_{H_k}}. \quad (2.11)$$

The Lagrangians and amplitudes for Z and $\tilde{\mu}_{L,R}$ exchange are given in appendix E, and the Lagrangians for the leptonic neutralino decays (2.2) are given in appendix C.

2.2 Squared amplitude

In order to calculate the squared amplitude for neutralino production (2.1) and decay (2.2), we use the spin density matrix formalism of [40, 41]. Following the detailed steps in appendix D, where we also give the production amplitudes, the squared amplitude in this formalism can be written as

$$|T|^2 = 2 |\Delta(\tilde{\chi}_j^0)|^2 \left(P \cdot D + \sum_{a=1}^3 \Sigma_P^a \Sigma_D^a \right), \quad (2.12)$$

with the propagator $\Delta(\tilde{\chi}_j^0)$ of the decaying neutralino, see eq. (D.7). Here P denotes the unpolarized production of the neutralinos and D the unpolarized decay. Σ_P^a and Σ_D^a are the corresponding polarized terms, and their product in eq. (2.12) describes the neutralino spin correlations between production and decay. With our definition of the spin density production matrix, eq. (D.8), Σ_P^3/P is the longitudinal polarization of $\tilde{\chi}_j^0$, Σ_P^1/P is the transverse polarization in the production plane, and Σ_P^2/P is the polarization perpendicular to the production plane. We give explicit expressions for the production terms P and Σ_P^a in the next section. The terms D and Σ_D^a for neutralino decay are given in appendix F.

2.3 Resonant contributions from Higgs exchange

The expansion coefficients of the squared neutralino production amplitude (2.12) subdivide into contributions from the Higgs resonances (res) and the continuum (cont), respectively,

$$P = P_{\text{res}} + P_{\text{cont}}, \quad \Sigma_P^a = \Sigma_{\text{res}}^a + \Sigma_{\text{cont}}^a, \quad a = 1, 2, 3. \quad (2.13)$$

The continuum contributions P_{cont} , Σ_{cont}^a are those from the non-resonant Z and $\tilde{\mu}_{L,R}$ exchange channels, with P_{cont} given in appendix E and Σ_{cont}^a in [41].

In order to analyze the dependence of the resonant contributions P_{res} and Σ_{res}^3 on the longitudinal μ^+ and μ^- beam polarizations \mathcal{P}_+ and \mathcal{P}_- , respectively, we can expand²

$$P_{\text{res}} = (1 + \mathcal{P}_+ \mathcal{P}_-) a_0 + (\mathcal{P}_+ + \mathcal{P}_-) a_1, \quad (2.14)$$

$$\Sigma_{\text{res}}^3 = (1 + \mathcal{P}_+ \mathcal{P}_-) b_0 + (\mathcal{P}_+ + \mathcal{P}_-) b_1. \quad (2.15)$$

Such an expansion proves to be useful for discussing CP properties. The coefficients a_0 and b_1 are CP-even, whereas a_1 and b_0 are CP-odd and vanish in the case of CP conservation [26]. They are given by

$$a_n = \sum_{k \leq l} (2 - \delta_{kl}) a_n^{kl}, \quad b_n = \sum_{k \leq l} (2 - \delta_{kl}) b_n^{kl}, \quad n = 0, 1; \quad (2.16)$$

with the sum over the contributions from the Higgs bosons H_k , H_l with $k, l = 1, 2, 3$, respectively, and

$$a_0^{kl} = \frac{s}{2} |\Delta_{(kl)}| \left[|c_\mu^+| |c_\chi^+| f_{ij} \cos(\delta_\mu^+ + \delta_\chi^+ + \delta_\Delta) - |c_\mu^+| |c_\chi^{\text{RL}}| m_i m_j \cos(\delta_\mu^+ + \delta_\chi^{\text{RL}} + \delta_\Delta) \right]_{(kl)}, \quad (2.17)$$

$$a_1^{kl} = \frac{s}{2} |\Delta_{(kl)}| \left[|c_\mu^-| |c_\chi^+| f_{ij} \cos(\delta_\mu^- + \delta_\chi^+ + \delta_\Delta) - |c_\mu^-| |c_\chi^{\text{RL}}| m_i m_j \cos(\delta_\mu^- + \delta_\chi^{\text{RL}} + \delta_\Delta) \right]_{(kl)}, \quad (2.18)$$

$$b_0^{kl} = -\frac{s}{4} |\Delta_{(kl)}| \left[|c_\mu^+| |c_\chi^-| \sqrt{\lambda_{ij}} \cos(\delta_\mu^+ + \delta_\chi^- + \delta_\Delta) \right]_{(kl)}, \quad (2.19)$$

$$b_1^{kl} = -\frac{s}{4} |\Delta_{(kl)}| \left[|c_\mu^-| |c_\chi^-| \sqrt{\lambda_{ij}} \cos(\delta_\mu^- + \delta_\chi^- + \delta_\Delta) \right]_{(kl)}, \quad (2.20)$$

We have defined the products of couplings, see eqs. (2.6) and (2.5), suppressing the neutralino indices i and j ,

$$c_{\alpha(kl)}^\pm = c_R^{H_k \alpha \alpha} c_R^{H_l \alpha \alpha^*} \pm c_L^{H_k \alpha \alpha} c_L^{H_l \alpha \alpha^*} = \left[|c_\alpha^\pm| \exp(i\delta_\alpha^\pm) \right]_{(kl)}, \quad \alpha = \mu, \chi, \quad (2.21)$$

$$c_{\chi(kl)}^{\text{RL}} = c_R^{H_k \chi \chi} c_L^{H_l \chi \chi^*} + c_L^{H_k \chi \chi} c_R^{H_l \chi \chi^*} = \left[|c_\chi^{\text{RL}}| \exp(i\delta_\chi^{\text{RL}}) \right]_{(kl)}, \quad (2.22)$$

the product of the Higgs boson propagators (2.11),

$$\Delta_{(kl)} = \Delta(H_k) \Delta(H_l)^* = \left[|\Delta| \exp(i\delta_\Delta) \right]_{(kl)}, \quad (2.23)$$

²The resonant contributions Σ_{res}^1 and Σ_{res}^2 to the transverse polarizations of the neutralino vanish for scalar Higgs bosons exchange in the s -channel.

and the kinematical functions $f_{ij} = \left(s - m_{\tilde{\chi}_i^0}^2 - m_{\tilde{\chi}_j^0}^2 \right) / 2$ and λ_{ij} , see eq. (D.13). Note that the coefficients b_0 and b_1 , which parametrize the neutralino polarization dependence in eq. (2.15), vanish at threshold as $\sqrt{\lambda_{ij}}$. The coefficients a_1 and b_1 contribute only for longitudinally polarized muon beams. These coefficients are products of the Higgs boson couplings to the muons and neutralinos. Our aim is to determine these coefficients using polarization asymmetries and the production cross section. Since a muon collider provides a good beam energy resolution it is the ideal tool to analyze their strong \sqrt{s} dependence.

We neglect interferences of the chirality violating Higgs exchange amplitudes with the chirality conserving continuum amplitudes, which are of order m_μ / \sqrt{s} . Note that contributions from H_1 exchange will be small far from its resonance.

2.4 Cross sections

We obtain cross sections and distributions by integrating the amplitude squared $|T|^2$ (2.12) over the Lorentz invariant phase space element $d\text{PS}$

$$d\sigma = \frac{1}{2s} |T|^2 d\text{PS}. \quad (2.24)$$

We use the narrow width approximation for the propagator of the decaying neutralino. Explicit formulas of the phase space for neutralino production (2.1) and decay (2.2), can be found, e.g., in [42]. The $\mu^+ \mu^-$ -spin averaged cross section for $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ neutralino production is

$$\sigma_{ij} = \frac{1}{(1 + \delta_{ij})} \frac{\sqrt{\lambda_{ij}}}{8\pi s^2} \bar{P}, \quad (2.25)$$

with the triangle function λ_{ij} (D.13), and the average over the neutralino production angles in the center-of-mass system,

$$\bar{P} = \frac{1}{4\pi} \int P d\Omega_{\tilde{\chi}_j^0}. \quad (2.26)$$

The integrated cross section for neutralino production (2.1) and subsequent leptonic decay $\tilde{\chi}_j^0 \rightarrow \ell^\pm \tilde{\ell}_n^\mp$ (2.2), with $n = R, L$ for $\ell = e, \mu$, and $n = 1, 2$ for $\ell = \tau$, is given by

$$\sigma_\ell^n = \frac{1}{(1 + \delta_{ij})} \frac{1}{64\pi^2} \frac{\sqrt{\lambda_{ij}}}{s^2} \frac{(m_{\tilde{\chi}_j^0}^2 - m_\ell^2)}{m_{\tilde{\chi}_j^0}^3 \Gamma_{\tilde{\chi}_j^0}} \bar{P} \cdot D = \sigma_{ij} \times \text{BR} \left(\tilde{\chi}_j^0 \rightarrow \ell^\pm \tilde{\ell}_n^\mp \right). \quad (2.27)$$

Explicit expressions for D are given in appendix F. Note that the integrated cross section σ_ℓ^n is independent of the neutralino polarizations.

2.5 Lepton energy distribution

The differential cross section $d\sigma$ (2.24), and thus the energy distribution of the lepton from the neutralino decay (2.2), depends on the longitudinal neutralino polarization. In the center-of-mass system, the kinematical limits of the energy of the decay lepton $\ell = e, \mu, \tau$ are

$$E_\ell^{\text{max}(\text{min})} = \hat{E}_\ell \pm \Delta_\ell, \quad (2.28)$$

with

$$\hat{E}_\ell = \frac{E_\ell^{\max} + E_\ell^{\min}}{2} = \frac{m_{\tilde{\chi}_j^0}^2 - m_\ell^2}{2m_{\tilde{\chi}_j^0}^2} E_{\tilde{\chi}_j^0}, \quad (2.29)$$

$$\Delta_\ell = \frac{E_\ell^{\max} - E_\ell^{\min}}{2} = \frac{m_{\tilde{\chi}_j^0}^2 - m_\ell^2}{2m_{\tilde{\chi}_j^0}^2} |\vec{p}_{\tilde{\chi}_j^0}|, \quad \ell = e, \mu, \tau. \quad (2.30)$$

Using the definition of the cross section (2.12), (2.27), and the explicit form of Σ_D^3 (F.7), the energy distribution of the decay lepton ℓ^\pm is [26, 27]

$$\frac{d\sigma_{\ell^\pm}^n}{dE_\ell} = \frac{\sigma_\ell^n}{2\Delta_\ell} \left[1 + \eta_{\ell^\pm} \eta_\ell^n \frac{\bar{\Sigma}_P^3 (E_\ell - \hat{E}_\ell)}{P \Delta_\ell} \right], \quad (2.31)$$

with $\eta_{\ell^\pm} = \mp 1$. The factor η_ℓ^n is a measure of parity violation in the neutralino decay. It is maximal $\eta_{e,\mu}^R = +1$ and $\eta_{e,\mu}^L = -1$ for the decay into $\tilde{e}_R, \tilde{\mu}_R$ and $\tilde{e}_L, \tilde{\mu}_L$, respectively, since the sleptons of the first two generations couple either purely left or right handed, if mixing is neglected. For the decay $\tilde{\chi}_j^0 \rightarrow \tau^\pm \tilde{\tau}_{1,2}^\mp$, the factor

$$\eta_\tau^n = \frac{|b_{nj}^{\tilde{\tau}}|^2 - |a_{nj}^{\tilde{\tau}}|^2}{|b_{nj}^{\tilde{\tau}}|^2 + |a_{nj}^{\tilde{\tau}}|^2}, \quad (2.32)$$

is generally smaller $|\eta_\tau^n| < 1$ due to stau mixing. The right and left $\tilde{\chi}_j^0 \tilde{\tau}_n \tau$ couplings $a_{nj}^{\tilde{\tau}}$ and $b_{nj}^{\tilde{\tau}}$ are defined in (G.2).

Further in eq. (2.31), the coefficients P and Σ_P^3 of the squared neutralino production amplitude (2.12) are averaged over the neutralino production solid angle, denoted by a bar in our notation (2.26). Due to the Majorana character of the neutralinos, the continuum contribution Σ_{cont}^3 (2.13) is forward-backward antisymmetric [43], and vanishes if integrated over the neutralino solid angle. However, the resonant contribution Σ_{res}^3 (2.13) from Higgs exchange is isotropic, thus

$$\bar{\Sigma}_P^3 = \frac{1}{4\pi} \int \Sigma_P^3 d\Omega_{\tilde{\chi}_j^0} = \Sigma_{\text{res}}^3. \quad (2.33)$$

In figure 3, we show the energy distributions (2.31) of the leptons ℓ^\pm from the decays $\tilde{\chi}_j^0 \rightarrow \ell^+ \tilde{\ell}_R^-$ and $\tilde{\chi}_j^0 \rightarrow \ell^- \tilde{\ell}_R^+$, for $\ell = e$ or μ . The cutoffs in the energy distributions of the primary leptons ℓ^+ and ℓ^- correspond to their kinematical limits, as given in eq. (2.28). We see the linear dependence of the distributions on the lepton energy. The slope of these distributions is proportional to the longitudinal neutralino polarization, and is solely due to the resonant Higgs contributions Σ_{res}^3 . In addition, we show in figure 3 the energy distributions from the secondary leptons ℓ_2^\pm from the subsequent decays $\tilde{\ell}_R^+ \rightarrow \ell_2^+ \tilde{\chi}_1^0$ and $\tilde{\ell}_R^- \rightarrow \ell_2^- \tilde{\chi}_1^0$.

Note that generally the parity conserving neutralino decays into the Z and the lightest Higgs boson, $\tilde{\chi}_j^0 \rightarrow Z \tilde{\chi}_k^0$, and $\tilde{\chi}_j^0 \rightarrow H_1 \tilde{\chi}_k^0$, respectively, cannot be used to analyze the neutralino polarization. The resulting energy distributions are flat, due to the Majorana

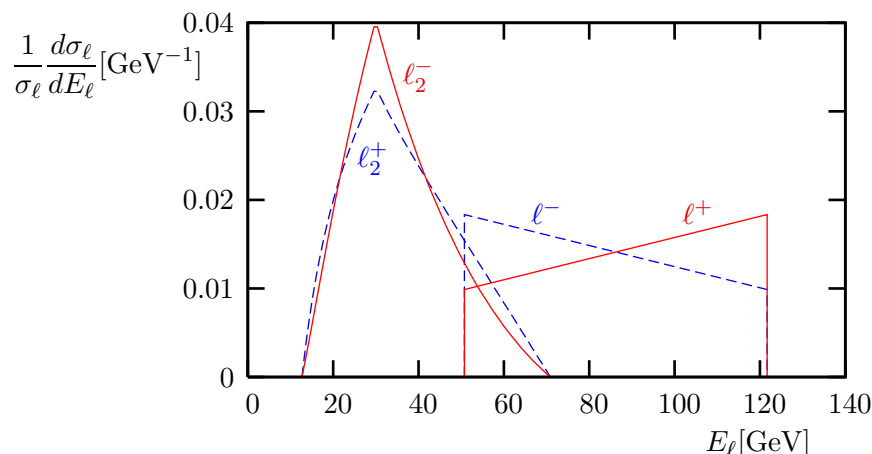


Figure 3: Normalized energy distributions of the primary ℓ^\pm and secondary leptons ℓ_2^\mp for neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ and subsequent decay chains $\tilde{\chi}_2^0 \rightarrow \ell^-\tilde{\ell}_R^+$, $\tilde{\ell}_R^+ \rightarrow \tilde{\chi}_1^0\ell_2^+$ (dashed) and $\tilde{\chi}_2^0 \rightarrow \ell^+\tilde{\ell}_R^-$, $\tilde{\ell}_R^- \rightarrow \tilde{\chi}_1^0\ell_2^-$ (solid) for $\ell = e$ or μ . Here $\mathcal{A}_\ell^R = -15\%$, corresponding to the masses of table 2, $\mathcal{P}_+ = \mathcal{P}_- = 0.3$, and $\sqrt{s} = 494$ GeV.

properties of the neutralinos. They imply that the left and right couplings obey $|O_{jk}^{\prime\prime R}| = |O_{jk}^{\prime\prime L}|$ (C.4), and $|c_R^{H_1\chi_j\chi_k}| = |c_L^{H_1\chi_j\chi_k}|$ (2.4). The decays are thus parity conserving, and therefore the parity violating factors analogous to η_τ^n (2.32) vanish.

3. Asymmetries for neutralino production and decay

In eqs. (2.14) and (2.15), we have expressed the resonant contributions P_{res} and Σ_{res}^3 to the spin density matrix elements for neutralino pair production in terms of the longitudinal muon beam polarizations. In order to experimentally determine the four different combinations of products of couplings a_0 , a_1 , b_0 , and b_1 , we define asymmetries of the neutralino production cross section, as well as asymmetries of the energy distributions of the decay leptons. Together with the neutralino production cross section, these coefficients can then be experimentally determined, and thus the Higgs couplings to muons and neutralinos.

3.1 Asymmetry of the neutralino production cross section

For the cross section of neutralino pair production $\sigma_{ij} = \sigma(\mu^+\mu^- \rightarrow \tilde{\chi}_i^0\tilde{\chi}_j^0)$, eq. (2.25), we define, for equal muon beam polarizations $\mathcal{P}_+ = \mathcal{P}_- \equiv \mathcal{P}$, the CP-odd asymmetry [38, 7]

$$\mathcal{A}_{\text{prod}}^{\text{pol}} = \frac{\sigma_{ij}(\mathcal{P}) - \sigma_{ij}(-\mathcal{P})}{\sigma_{ij}(\mathcal{P}) + \sigma_{ij}(-\mathcal{P})}. \tag{3.1}$$

$\mathcal{A}_{\text{prod}}^{\text{pol}}$ is sensitive to the CP phases of the Higgs boson couplings to the neutralinos and to the muons. Denoting by \tilde{T} the naïve time reversal $t \rightarrow -t$, which inverts momenta and spins without exchanging initial and final particles, the asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ is also $\text{CPT}\tilde{T}$ -odd. Thus the asymmetry is due to the interference of the CP phases with the absorptive phases from the transition amplitudes. The absorptive phases are also called strong phases, and

originate from intermediate particles which go on-shell. The asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ is therefore sensitive to the CP phases of the Higgs boson couplings, as well as to the phases of the Higgs propagators.

In the Higgs decoupling limit [15], the heavy neutral Higgs bosons are nearly mass degenerate. Thus a mixing of the CP-even and CP-odd Higgs states H and A can be resonantly enhanced, and large CP-violating Higgs couplings can be obtained [8, 9]. In addition, CP phases in the Higgs sector lead to a larger splitting of the mass eigenstates H_2 and H_3 . This in general tends to increase the phase difference between the Higgs propagators, giving rise to larger absorptive phases in the transition amplitudes.

Using the definitions of the neutralino production cross section σ_{ij} (2.25), and of the P term (2.14), we obtain

$$\mathcal{A}_{\text{prod}}^{\text{pol}} = \frac{2\mathcal{P}a_1}{(1 + \mathcal{P}^2)a_0 + \bar{P}_{\text{cont}}}. \quad (3.2)$$

We can thus employ the asymmetry to determine the CP-odd coefficient a_1 . The maximum absolute value of the asymmetry depends on the beam polarization \mathcal{P}

$$\mathcal{A}_{\text{prod}}^{\text{pol, max}} = \frac{2\mathcal{P}}{1 + \mathcal{P}^2}, \quad (3.3)$$

which follows from eq. (3.2), for vanishing continuum contributions $\bar{P}_{\text{cont}} = 0$.

Note that the coefficient a_0 can be obtained from the neutralino production cross section σ_{ij} (2.25). For example, for unpolarized beams, $\mathcal{P}_+ = \mathcal{P}_- = \mathcal{P} = 0$,

$$a_0 = \sigma_{ij} \frac{8\pi s^2}{\sqrt{\lambda_{ij}}} (1 + \delta_{ij}). \quad (3.4)$$

Here we assume that the continuum contributions \bar{P}_{cont} (2.13) to the cross section σ_{ij} are already subtracted, e.g. through an extrapolation of σ_{ij} around the resonances [22], and/or by neutralino cross section measurements at the International Linear Collider (ILC) [44, 45].

3.2 Asymmetries of the lepton energy distribution

The longitudinal neutralino polarization is also sensitive to the Higgs interference in the production $\mu^+ \mu^- \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$. The neutralino polarization can be analyzed by the subsequent decays $\tilde{\chi}_j^0 \rightarrow \ell^\pm \tilde{\ell}_{R,L}^\mp$, with $\ell = e, \mu$, and $\tilde{\chi}_j^0 \rightarrow \tau^\pm \tilde{\tau}_{1,2}^\mp$. In section 2.5, we have shown that the slope of the lepton energy distribution, see figure 3, is proportional to the averaged longitudinal neutralino polarization $\Sigma_{\text{res}}^3 / \bar{P}$. The polarization can be determined by the the energy distribution asymmetry [26]

$$\begin{aligned} \mathcal{A}_{\ell^\pm}^n &= \frac{\Delta\sigma_{\ell^\pm}^n}{\sigma_{\ell^\pm}^n} = \frac{1}{2} \eta_\ell^n \eta_{\ell^\pm} \frac{\Sigma_{\text{res}}^3}{\bar{P}} \\ &= \frac{1}{2} \eta_\ell^n \eta_{\ell^\pm} \frac{(1 + \mathcal{P}_+ \mathcal{P}_-) b_0 + (\mathcal{P}_+ + \mathcal{P}_-) b_1}{P_{\text{res}} + \bar{P}_{\text{cont}}}, \end{aligned} \quad (3.5)$$

with $\Delta\sigma_{\ell^\pm}^n = \sigma_{\ell^\pm}^n(E_\ell > \hat{E}_\ell) - \sigma_{\ell^\pm}^n(E_\ell < \hat{E}_\ell)$, and $n = R, L$ for $\ell = e, \mu$, and $n = 1, 2$ for $\ell = \tau$. Here we have used the explicit formula for the energy distribution of the decay

lepton ℓ^\pm (2.31), with $\bar{\Sigma}_P^3 = \Sigma_{\text{res}}^3$ (2.33). The continuum contributions to the neutralino polarization $\bar{\Sigma}_{\text{cont}}^3 = 0$ vanish due to the Majorana properties of the neutralinos, see section 2.5.

The average neutralino polarization is thus solely due to the Higgs exchange, and receives CP-even and CP-odd contributions, proportional to b_1 and b_0 , respectively. In order to separate these coefficients we define the polarization asymmetries

$$\mathcal{A}_{\ell^\pm}^{\text{pol},n} = \frac{\Delta\sigma_{\ell^\pm}^n(\mathcal{P}) - \Delta\sigma_{\ell^\pm}^n(-\mathcal{P})}{\sigma_{\ell^\pm}^n(\mathcal{P}) + \sigma_{\ell^\pm}^n(-\mathcal{P})} = \eta_\ell^n \eta_{\ell^\pm} \frac{\mathcal{P}b_1}{(1+\mathcal{P}^2)a_0 + \bar{P}_{\text{cont}}}, \quad (3.6)$$

$$\mathcal{A}'_{\ell^\pm}{}^{\text{pol},n} = \frac{\Delta\sigma_{\ell^\pm}^n(\mathcal{P}) + \Delta\sigma_{\ell^\pm}^n(-\mathcal{P})}{\sigma_{\ell^\pm}^n(\mathcal{P}) + \sigma_{\ell^\pm}^n(-\mathcal{P})} = \frac{1}{2} \eta_\ell^n \eta_{\ell^\pm} \frac{(1+\mathcal{P}^2)b_0}{(1+\mathcal{P}^2)a_0 + \bar{P}_{\text{cont}}}, \quad (3.7)$$

for equal muon beam polarizations $\mathcal{P}_+ = \mathcal{P}_- \equiv \mathcal{P}$.

The slepton from the neutralino decay, $\tilde{\chi}_j^0 \rightarrow \ell^\pm \tilde{\ell}_n^\mp$, subsequently decays into a neutralino and a secondary lepton. The primary and secondary leptons have to be distinguished from each other, for example, by using their different energy distributions, see figure 3. However, the largest part of that irreducible background from the secondary lepton cancels in forming the charge conjugated asymmetries [26]

$$\mathcal{A}_\ell^{\text{pol},n} = \frac{1}{2} \left(\mathcal{A}_{\ell^+}^{\text{pol},n} - \mathcal{A}_{\ell^-}^{\text{pol},n} \right) = \eta_\ell^n \frac{\mathcal{P}b_1}{(1+\mathcal{P}^2)a_0 + \bar{P}_{\text{cont}}}, \quad (3.8)$$

$$\mathcal{A}'_\ell{}^{\text{pol},n} = \frac{1}{2} \left(\mathcal{A}'_{\ell^+}{}^{\text{pol},n} - \mathcal{A}'_{\ell^-}{}^{\text{pol},n} \right) = \frac{1}{2} \eta_\ell^n \frac{(1+\mathcal{P}^2)b_0}{(1+\mathcal{P}^2)a_0 + \bar{P}_{\text{cont}}}. \quad (3.9)$$

Due to the pure left or right coupling structure of the neutralinos to the selectrons and smuons, the asymmetries for the decay into $\tilde{e}_R, \tilde{\mu}_R$ and $\tilde{e}_L, \tilde{\mu}_L$, respectively, have opposite sign:

$$\mathcal{A}_\ell^{\text{pol},R} = -\mathcal{A}_\ell^{\text{pol},L}, \quad \mathcal{A}'_\ell{}^{\text{pol},R} = -\mathcal{A}'_\ell{}^{\text{pol},L}, \quad \ell = e, \mu, \quad (3.10)$$

which follows from $\eta_{e,\mu}^R = +1$ and $\eta_{e,\mu}^L = -1$. The asymmetries for the decay into $\tilde{\tau}_1$ and $\tilde{\tau}_2$ are always smaller than those for the decays into a selectron or smuon,

$$\mathcal{A}_\tau^{\text{pol},n} = \eta_\tau^n \mathcal{A}_\ell^{\text{pol},R}, \quad \mathcal{A}'_\tau{}^{\text{pol},n} = \eta_\tau^n \mathcal{A}'_\ell{}^{\text{pol},R}, \quad n = 1, 2, \quad \ell = e, \mu, \quad (3.11)$$

with $|\eta_\tau^n| \leq 1$, due to mixing in the stau sector, see eq. (2.32).

The CP-even asymmetry $\mathcal{A}_\ell^{\text{pol},n}$ is due to the correlation between the longitudinal polarizations of the initial muons and final neutralinos, see appendix B. Large values of the CP-even asymmetry $\mathcal{A}_\ell^{\text{pol},n}$ can be obtained when both Higgs resonances are nearly degenerate, and if their amplitudes are of the same magnitude. However, a scalar-pseudoscalar mixing in the presence of CP phases will in general increase the mass splitting of the Higgs bosons, and the reduced overlap of the Higgs resonances also reduces the CP-even asymmetry $\mathcal{A}_\ell^{\text{pol},n}$.

The CP-odd asymmetry $\mathcal{A}'_\ell{}^{\text{pol},n}$ vanishes for CP-conserving Higgs couplings. Similarly to the CP-odd polarization asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ (3.2) for neutralino production, the decay asymmetry $\mathcal{A}'_\ell{}^{\text{pol},n}$ is approximately maximal if the Higgs mixing is resonantly enhanced. As pointed out earlier, this can happen naturally in the Higgs decoupling limit.

$M_{H^\pm} = 500 \text{ GeV}$	$\tan \beta = 10$	$ A = 1 \text{ TeV}$	$\phi_A = 0.2\pi$
$M_{\text{SUSY}} = 500 \text{ GeV}$	$\mu = 400 \text{ GeV}$	$M_2 = 300 \text{ GeV}$	$m_0 = 100 \text{ GeV}$

Table 1: SUSY parameters for the benchmark scenario $\text{CP}\chi$. The slepton masses are parametrized by m_0 , the squark masses by M_{SUSY} .

$M_{H_1} = 126.0 \text{ GeV}$	$m_{\tilde{\chi}_1^0} = 147 \text{ GeV}$	$m_{\tilde{e}_R} = 180 \text{ GeV}$	$\text{BR}(\tilde{\chi}_2^0 \rightarrow e^+ \tilde{e}_R^-) = 6.4\%$
$M_{H_2} = 492.8 \text{ GeV}$	$m_{\tilde{\chi}_2^0} = 275 \text{ GeV}$	$m_{\tilde{e}_L} = 289 \text{ GeV}$	$\text{BR}(\tilde{\chi}_2^0 \rightarrow \tau^+ \tilde{\tau}_1^-) = 23\%$
$M_{H_3} = 493.6 \text{ GeV}$	$m_{\tilde{\chi}_3^0} = 405 \text{ GeV}$	$m_{\tilde{\tau}_1} = 178 \text{ GeV}$	$\text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z) = 9.1\%$
$\Gamma_{H_2} = 0.97 \text{ GeV}$	$m_{\tilde{\chi}_1^\pm} = 274 \text{ GeV}$	$m_{\tilde{\tau}_2} = 290 \text{ GeV}$	$\text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 H_1) = 18\%$
$\Gamma_{H_3} = 0.93 \text{ GeV}$	$m_{\tilde{\chi}_2^\pm} = 433 \text{ GeV}$	$m_{\tilde{\nu}} = 277 \text{ GeV}$	$\eta_\tau^1 = -0.53$

Table 2: SUSY masses, widths, and branching ratios for the benchmark scenario $\text{CP}\chi$, evaluated with `FeynHiggs 2.5.1` [16, 17].

4. Numerical results

We analyze numerically the CP-odd asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ (3.1) for neutralino production $\mu^+ \mu^- \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$. For the subsequent decays, $\tilde{\chi}_2^0 \rightarrow e \tilde{e}_R$ and $\tilde{\chi}_2^0 \rightarrow \tau \tilde{\tau}_1$, we study the CP-even and CP-odd polarization asymmetries $\mathcal{A}_e^{\text{pol},R}$, $\mathcal{A}_\tau^{\text{pol},1}$ (3.8), and $\mathcal{A}_e^{\text{pol},R}$ (3.9), respectively. The feasibility of measuring the asymmetries also depends on the neutralino production cross section and decay branching ratios, which we discuss in detail.

We induce CP violation in the Higgs sector by a non-vanishing phase ϕ_A of the common trilinear scalar coupling parameter $A_t = A_b = A_\tau \equiv |A| \exp(i\phi_A)$ for the third generation fermions. This assignment is also compatible with the bounds on CP-violating phases from experiments on electric dipole moments (EDMs) [46–49]. We assume a CP-conserving gaugino sector, i.e., we keep the gaugino mass parameters M_1 , M_2 , and the Higgs mass parameter μ real. For the calculation of the Higgs masses, widths and couplings, we use the program `FeynHiggs 2.5.1` [16, 17], see also appendix A. For the branching ratios and decay width of the neutralino, we include the two-body decays [42]

$$\tilde{\chi}_2^0 \rightarrow \ell + \tilde{\ell}_n, \quad \nu_\ell + \tilde{\nu}_\ell, \quad \tilde{\chi}_1^0 + Z, \quad \tilde{\chi}_1^0 + H_1, \quad (4.1)$$

with $n = R, L$ for $\ell = e, \mu$, and $n = 1, 2$ for $\ell = \tau$. We neglect three-body decays. In order to enable the leptonic neutralino decays $\tilde{\chi}_2^0 \rightarrow \ell \tilde{\ell}_n$, we need light sleptons. We parametrize their masses by m_0 and M_2 , which enter in the approximate solutions to the renormalization group equations, see appendix F. We parametrize the diagonal entries of the squark mass matrices by the common SUSY scale parameter $M_{\text{SUSY}} = M_{\tilde{Q}_3} = M_{\tilde{U}_3} = M_{\tilde{D}_3}$. Finally, in order to reduce the number of parameters, we assume the GUT relation for the gaugino mass parameters $M_1 = 5/3 M_2 \tan^2 \theta_W$.

We center our numerical discussion around scenario $\text{CP}\chi$, defined in table 1. Inspired by the benchmark scenario CPX [50] for studying enhanced CP-violating Higgs-mixing

phenomena, we set $|A| = 2M_{\text{SUSY}} = 1 \text{ TeV}$, $M_3 = 800 \text{ GeV}$, and a non-vanishing phase $\phi_A = 0.2\pi$. We thus obtain large contributions from the trilinear coupling parameter A of the third generation to the Higgs sector, both CP-conserving and CP-violating. In contrast to the CPX scenario, we do not need large values of $|\mu|$ to obtain large CP violating effects, see discussion in section 4.2. We choose $\mu = 400 \text{ GeV}$ and $M_2 = 300 \text{ GeV}$ of similar size to enhance the branching ratios of the Higgs bosons into neutralinos, which are large only for mixed neutralinos. We fix $\tan\beta = 10$, since the Higgs boson decays into neutralinos are most relevant for intermediate values of $\tan\beta$. Smaller values of $\tan\beta$ favor the $t\bar{t}$ decay channel, while larger values enhance decays into $b\bar{b}$ and $\tau\bar{\tau}$. We give the masses of the Higgs bosons, the charginos, neutralinos, light sleptons, and the widths of the Higgs bosons for scenario CP χ in table 2, where we also list the branching ratios for the decaying neutralino. We choose longitudinal muon beam polarizations of $\mathcal{P}_+ = \mathcal{P}_- = \mathcal{P} = \pm 0.3$, which should be feasible at a muon collider [37].

4.1 \sqrt{s} dependence

For the scenario CP χ , we analyze the dependence of the asymmetries and the cross section on the center-of-mass energy \sqrt{s} . The CP-even and CP-odd observables exhibit a characteristic \sqrt{s} dependence, mainly given by the product of Higgs boson propagators $\Delta_{(kl)}$, see eq. (2.23). A muon collider will have a very precise beam energy resolution, and thus enables detailed line-shape scans.

In figure 4(a), we show the CP-odd polarization asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ for neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ as a function of \sqrt{s} around the heavy Higgs resonances H_2 and H_3 . At the peak value, $\sqrt{s} = (M_{H_2} + M_{H_3})/2 \approx 493 \text{ GeV}$, the interference of the two nearly degenerate Higgs bosons is maximal, leading to an asymmetry of up to $\mathcal{A}_{\text{prod}}^{\text{pol}} = 30\%$. The asymmetry measures the difference of the neutralino production cross section $\sigma_{12}(\mathcal{P})$ for equal positive and negative muon beam polarizations $\mathcal{P} = \pm 0.3$, which we show in figure 4(b). We also observe that the splitting of the two resonances is increased in the presence of CP-violating phases. For $\phi_A = 0.2\pi$, the two resonances are clearly visible in the line shape of the cross section σ_{12} , whereas it assumes a single resonance form for $\phi_A = 0$, where the Higgs bosons are extremely degenerate, see figure 4(b).

The Higgs boson interference in neutralino production also leads to CP-even and CP-odd contributions to the average longitudinal neutralino polarizations. In order to analyze the $\tilde{\chi}_2^0$ polarization, we discuss the CP-even asymmetry $\mathcal{A}_e^{\text{pol},R}$ and the CP-odd asymmetry $\mathcal{A}_e^{\text{pol},R}$ of the leptonic energy distributions for the neutralino decay. For simplicity, we discuss only the decay into a right selectron $\tilde{\chi}_2^0 \rightarrow e\tilde{e}_R$. The same asymmetries are obtained for the decay into a right smuon $\tilde{\chi}_2^0 \rightarrow \mu\tilde{\mu}_R$. For the neutralino decay into a tau, $\tilde{\chi}_2^0 \rightarrow \tau\tilde{\tau}_1$, the corresponding asymmetries are obtained from eq. (3.11).

For the decay $\tilde{\chi}_2^0 \rightarrow e\tilde{e}_R$, we show the \sqrt{s} dependence of the CP- and CP \tilde{T} -even asymmetry $\mathcal{A}_e^{\text{pol},R}$ in figure 4(c) for $\phi_A = 0$ and $\phi_A = 0.2\pi$. The CP- and CP \tilde{T} -odd asymmetry $\mathcal{A}_e^{\text{pol},R}$ is shown in figure 4(d). The phase ϕ_A tends to increase the mass splitting of the Higgs resonances. Their overlap is now reduced, leading in general to a suppression of the CP-even asymmetry $\mathcal{A}_e^{\text{pol},R}$, in particular at the mean energy of the

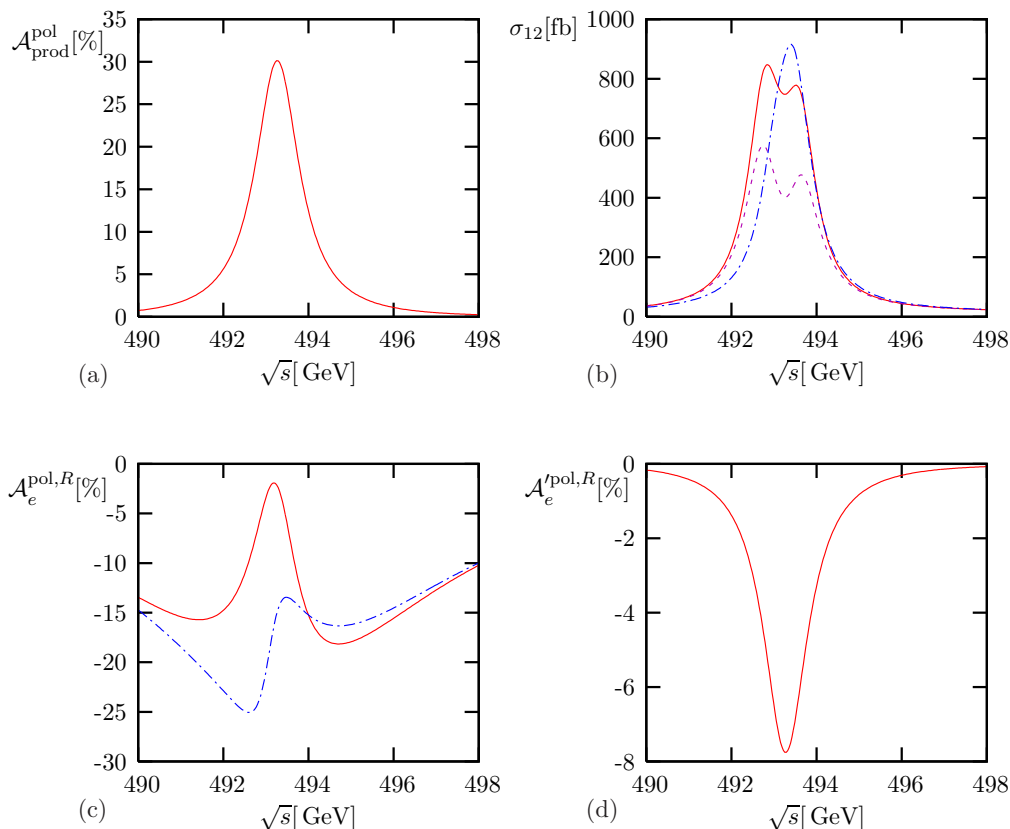


Figure 4: \sqrt{s} dependence of (a) the CP-odd production asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$, eq. (3.1), and (b) the cross section σ_{12} for neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$. For the subsequent decay, $\tilde{\chi}_2^0 \rightarrow e\tilde{e}_R$, in (c) the CP-even polarization asymmetry $\mathcal{A}_e^{\text{pol},R}$, eq. (3.8), and in (d) the CP-odd polarization asymmetry $\mathcal{A}_e^{\prime\text{pol},R}$, eq. (3.9), for the SUSY parameters as given in table 1. The longitudinal beam polarizations are $\mathcal{P}_+ = \mathcal{P}_- = \mathcal{P} = +0.3$ (solid), in (b) $\mathcal{P} = -0.3$ (dashed), and in (b,c) $\mathcal{P} = +0.3$ with $\phi_A = 0$ (dot-dashed).

resonances $\sqrt{s} = (M_{H_2} + M_{H_3})/2$, see figure 4(c). On the contrary, the larger Higgs splitting increases the CP-odd asymmetries $\mathcal{A}_e^{\text{pol},R}$ and $\mathcal{A}_{\text{prod}}^{\text{pol}}$.

All asymmetries for production and decay vanish asymptotically far from the resonance region. The continuum contributions from selectron and Z exchange to the difference of the cross sections and to the average neutralino polarization cancel in the numerator, but contribute in the denominator of the corresponding asymmetries, see their definitions in section 3.

In the following sections, we analyze the dependence of the production cross section and the asymmetries on $|A|$ and ϕ_A , and finally on M_2 and μ , fixing all remaining parameters to those of scenario $\text{CP}\chi$. We fix the center-of-mass energy to $\sqrt{s} = (M_{H_2} + M_{H_3})/2$, where we expect the largest CP-odd asymmetries $\mathcal{A}_{\text{prod}}^{\text{pol}}$ and $\mathcal{A}_e^{\text{pol},R}$, see figures 4(a) and (d), respectively. For consistency, we also choose $\sqrt{s} = (M_{H_2} + M_{H_3})/2$ for the discussion of the CP-even asymmetry $\mathcal{A}_e^{\text{pol},R}$, although it is generally suppressed at this value if CP is violated.

4.2 $|A|$ and ϕ_A dependence

We analyze the dependence of the CP-asymmetries on the phase ϕ_A of the trilinear coupling A , which is the only source of CP violation in our study. The CP-odd asymmetries, $\mathcal{A}_{\text{prod}}^{\text{pol}}$ and $\mathcal{A}_e^{\text{pol},R}$, see figure 5(a), are approximately maximal, if the mixing of the Higgs states is resonantly enhanced. This is naturally achieved when the diagonal elements $m_H^2 - \hat{\Sigma}_{\text{HH}}(s)$ and $m_A^2 - \hat{\Sigma}_{\text{AA}}(s)$ of the Higgs mass matrix \mathbf{M} , eq. (2.8), are equal, provided the corresponding imaginary part is small. In our scenario, where decays into heavy squarks are not kinematically allowed, this condition is roughly fulfilled for $\phi_A \simeq 0.2\pi$. We interpret this condition as a level crossing of the CP-eigenstates H and A , when $m_H^2 - m_A^2 - \text{Re}[\hat{\Sigma}_{\text{HH}}(s) - \hat{\Sigma}_{\text{AA}}(s)]$ changes sign [10]. The mass difference of the physical Higgs boson masses, however, is typically increased by the H - A mixing, as can be observed from figure 5(c). A splitting of the order of the Higgs widths $\Gamma_{H_{2,3}}$, shown in figure 5(d), leads to large absorptive phases, which are necessary for the presence of CPT-odd observables. The increased Higgs mass splitting for non-vanishing phases leads, however, in general to lower peak cross sections $\sigma_{12}(\mathcal{P})$, which we show in figure 5(b), both for positive and negative beam polarizations $\mathcal{P} = \pm 0.3$.

The asymmetries and cross section for negative ϕ_A can be obtained from symmetry considerations. Since the complex trilinear coupling A is the only source of CP violation in our analysis, the CP-odd asymmetries $\mathcal{A}_{\text{prod}}^{\text{pol}}$ and $\mathcal{A}_e^{\text{pol},R}$ must be odd with respect to the transformation $\phi_A \rightarrow -\phi_A$, while the CP-even asymmetry $\mathcal{A}_e^{\text{pol},R}$ must be even. Consequently, the cross section transforms as $\sigma_{12}(\mathcal{P}) \rightarrow \sigma_{12}(-\mathcal{P})$.

In figure 6, we show contour lines of the cross section and the asymmetries in the ϕ_A - $|A|$ plane. The largest CP-odd asymmetries $\mathcal{A}_{\text{prod}}^{\text{pol}}$ and $\mathcal{A}_e^{\text{pol},R}$ are obtained for $|A| \approx 2M_{\text{SUSY}} = 1 \text{ TeV}$. For larger values of $|A|$, the lighter stops become kinematically accessible and H_2 decays dominantly into $\tilde{t}_1^+ \tilde{t}_1^-$ pairs, which leads to a suppression of the neutralino production cross section. We therefore restrict our discussion to $|A| < 1.2 \text{ TeV}$.

As we have observed in figure 5(a) of the preceding paragraph, the CP-even asymmetry $\mathcal{A}_e^{\text{pol},R}$, figure 6(c), is in general larger in the CP-conserving limit. The maximum of the asymmetry is also obtained for $A \approx \pm 800 \text{ GeV}$. However, this is rather coincidental, and is due to the exact degeneracy of the Higgs bosons H and A .

Note that large resonant mixing is possible without requiring large values of $|\mu|$. The CP-violating scalar-pseudoscalar self energy transitions are proportional to the amount of CP violation in the squark sector, described by the quantity

$$\frac{3}{16\pi^2} \frac{\text{Im}(A_f \mu)}{m_{\tilde{f}_2}^2 - m_{\tilde{f}_1}^2}, \tag{4.2}$$

with $f = t, b$ [6, 8, 28]. However we obtain large H - A mixing for moderate values of μ , since, in the Higgs decoupling limit $\text{Im}\hat{\Sigma}_{\text{HH}}(s) \simeq \text{Im}\hat{\Sigma}_{\text{AA}}(s)$ for energies below the threshold of heavy squark pair production. Therefore, the conditions for maximally resonance enhanced mixing discussed in this section may be fulfilled for moderate values of $\hat{\Sigma}_{\text{HA}}(s)$.

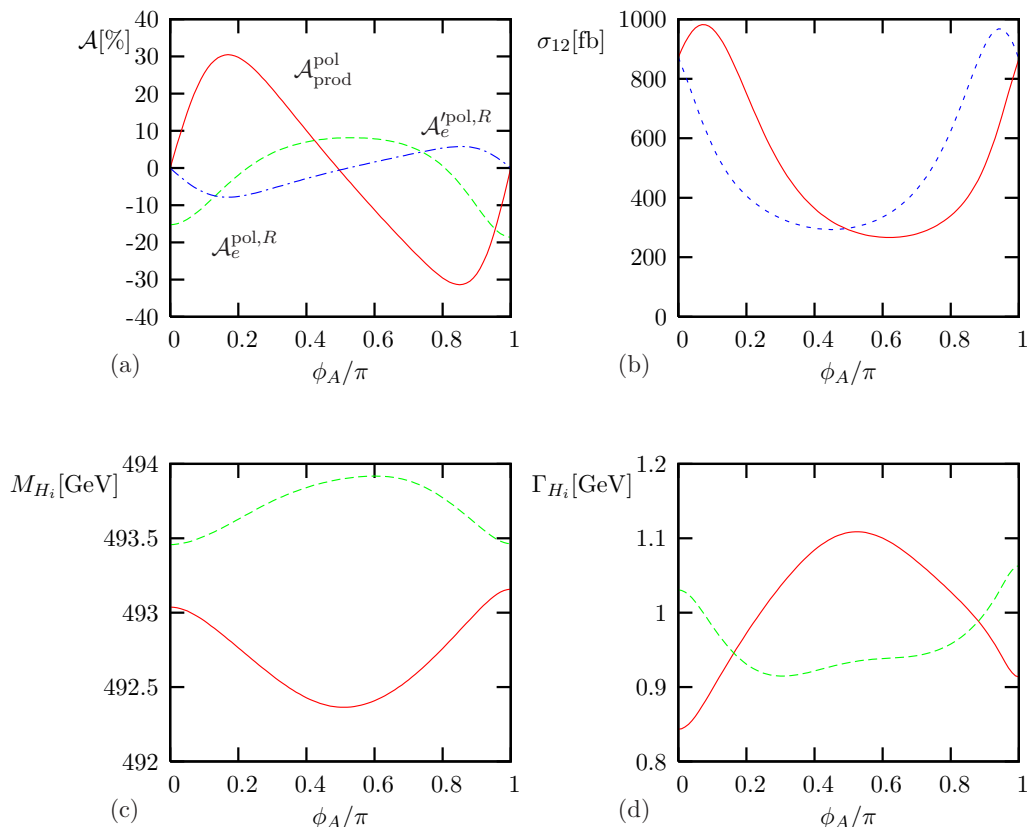


Figure 5: Phase dependence of (a) the CP-odd polarization asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ (solid), eq. (3.1), for neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ at $\sqrt{s} = (M_{H_2} + M_{H_3})/2$, and for the subsequent decay $\tilde{\chi}_2^0 \rightarrow e\tilde{e}_R$ the CP-even polarization asymmetry $\mathcal{A}_e^{\text{pol},R}$ (dashed), eq. (3.8), and the CP-odd polarization asymmetry $\mathcal{A}_e^{\prime\text{pol},R}$ (dot-dashed), eq. (3.9). In (b) the cross section σ_{12} for neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ with longitudinal beam polarizations $\mathcal{P}_+ = \mathcal{P}_- = \mathcal{P} = +0.3$ (solid), and $\mathcal{P} = -0.3$ (dotted). In (c) the Higgs masses M_{H_i} and (d) the Higgs widths Γ_{H_i} , for $i = 2$ (solid), and $i = 3$ (dashed). The SUSY parameters are given in table 1.

4.3 μ and M_2 dependence

The couplings of the Higgs bosons to the neutralinos strongly depend on the gaugino-higgsino composition of the neutralinos, which are mainly determined by the values of μ and M_2 . For neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$, we show the CP-odd polarization asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ (3.1) in the μ - M_2 plane in figure 7(a). For $\mathcal{P} = \pm 0.3$, the maximum absolute value of the asymmetry would be $\mathcal{A}_{\text{prod}}^{\text{pol,max}} \approx 55\%$, as follows from eq. (3.3). We observe in figure 7(a) that the asymmetry reaches 40% near the neutralino production threshold, where the coefficient a_1 , see eq. (2.18), receives large spin-flip contributions. For smaller μ and M_2 the Higgs boson widths are increased, since decay channels into light neutralinos and charginos open. This results in a larger overlap of the Higgs resonances, which reduces the absorptive phases, and consequently suppresses the CP \tilde{T} -odd asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$. In the upper left corner of figure 7(a), the asymmetry changes sign due to a level

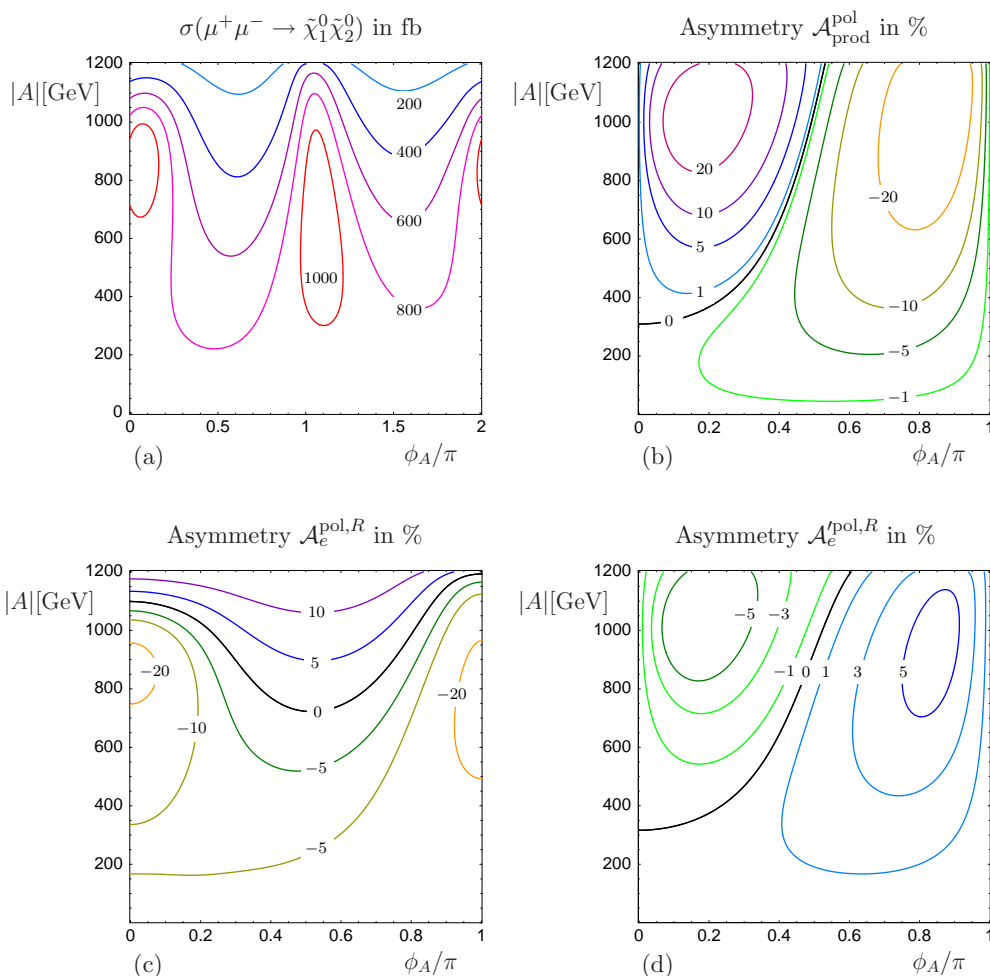


Figure 6: Contour lines in the ϕ_A - $|A|$ plane for (a) the cross section for neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$, (b) the CP-odd polarization asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$, and for the subsequent decay $\tilde{\chi}_2^0 \rightarrow e\tilde{e}_R$, (c) the CP-even polarization asymmetry $\mathcal{A}_e^{\text{pol},R}$, eq. (3.8), and (d) the CP-odd polarization asymmetry $\mathcal{A}_e^{\text{pol},R}$, eq. (3.9), at $\sqrt{s} = (M_{H_2} + M_{H_3})/2$ with longitudinally polarized beams $\mathcal{P}_- = \mathcal{P}_+ = \pm 0.3$. The SUSY parameters are given in table 1.

crossing of the two neutralinos $\tilde{\chi}_2^0$ and $\tilde{\chi}_3^0$.

In figure 8(a), we show the cross section σ_{12} for neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$. In the mixed region $|\mu| \simeq M_2$, where the Higgs-neutralino couplings are larger, the cross section reaches up to $\sigma_{12} \approx 1500$ fb. In addition, since H_2 and H_3 are mixed CP-eigenstates, there is no p -wave suppression. However, due to the Majorana nature of the neutralinos, the continuum contribution from $\tilde{\mu}$ and Z exchange to the cross section is p -wave suppressed [51]. It is thus negligible near threshold, and reaches 150 fb only for $\mu \lesssim 150$ GeV. In figure 8(b), we show the branching ratio for the neutralino decay $\tilde{\chi}_2^0 \rightarrow e^+\tilde{e}_R^-$. The decay fraction is reduced in the upper right corner, since the channels $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 Z$, and $\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 H_1$, open. In particular, the branching ratio into the lightest Higgs boson can be

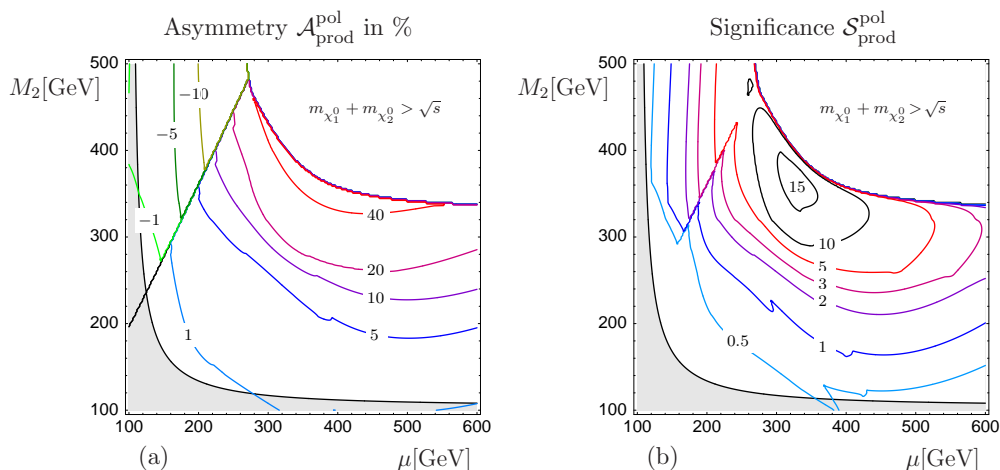


Figure 7: Neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ at $\sqrt{s} = (M_{H_2} + M_{H_3})/2$ with longitudinally polarized beams $\mathcal{P}_- = \mathcal{P}_+ = -0.3$. Contour lines in the μ - M_2 plane for (a) the CP-odd production asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ (3.1), and (b) the significance $\mathcal{S}_{\text{prod}}^{\text{pol}}$ (H.1), with $\mathcal{L} = 1 \text{ fb}^{-1}$, for the SUSY parameters as given in table 1. The corresponding cross section $\sigma(\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ is given in figure 8. The shaded area is excluded by requiring $m_{\chi_{\pm 1}} > 103$ GeV.

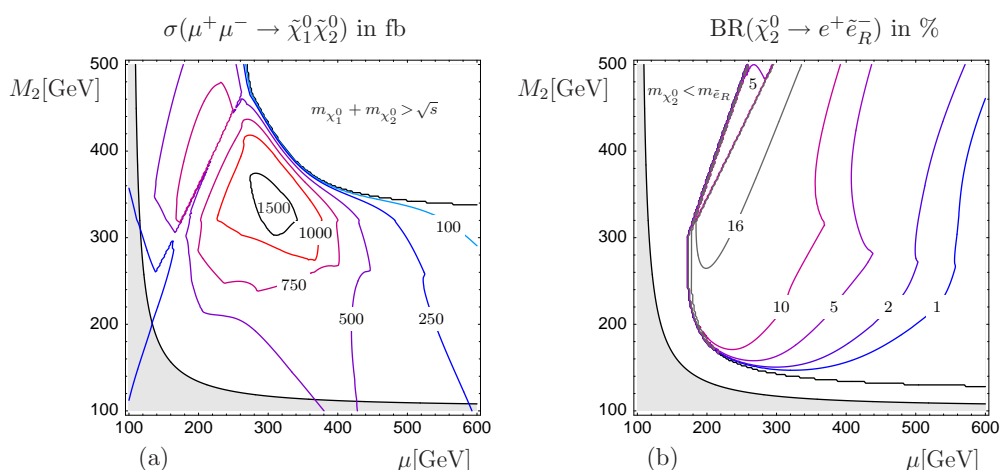


Figure 8: Contour lines in the μ - M_2 of (a) the cross section $\sigma_{12} = \sigma(\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ at $\sqrt{s} = (M_{H_2} + M_{H_3})/2$ with longitudinally polarized beams $\mathcal{P}_- = \mathcal{P}_+ = 0.3$, (b) the branching ratio $\text{BR}(\tilde{\chi}_2^0 \rightarrow e^+\tilde{e}_R^-)$, for the SUSY parameters as given in table 1. The shaded area is excluded by requiring $m_{\chi_{\pm 1}} > 103$ GeV.

$\text{BR}(\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 H_1) > 60\%$, for $M_2 \gtrsim 400$ GeV.

For the neutralino decay $\tilde{\chi}_2^0 \rightarrow e\tilde{e}_R$, we show the polarization asymmetries $\mathcal{A}_e^{\text{pol},R}$ and $\mathcal{A}_e^{\text{pol},L}$ in figure 9(a) and (b), respectively. As discussed before, the CP-even asymmetry $\mathcal{A}_e^{\text{pol},R}$ is suppressed by CP-violating effects due to the smaller overlap of the resonances at $\sqrt{s} = (M_{H_2} + M_{H_3})/2$. Therefore we only find large values of $\mathcal{A}_e^{\text{pol},R}$ for light neutralinos

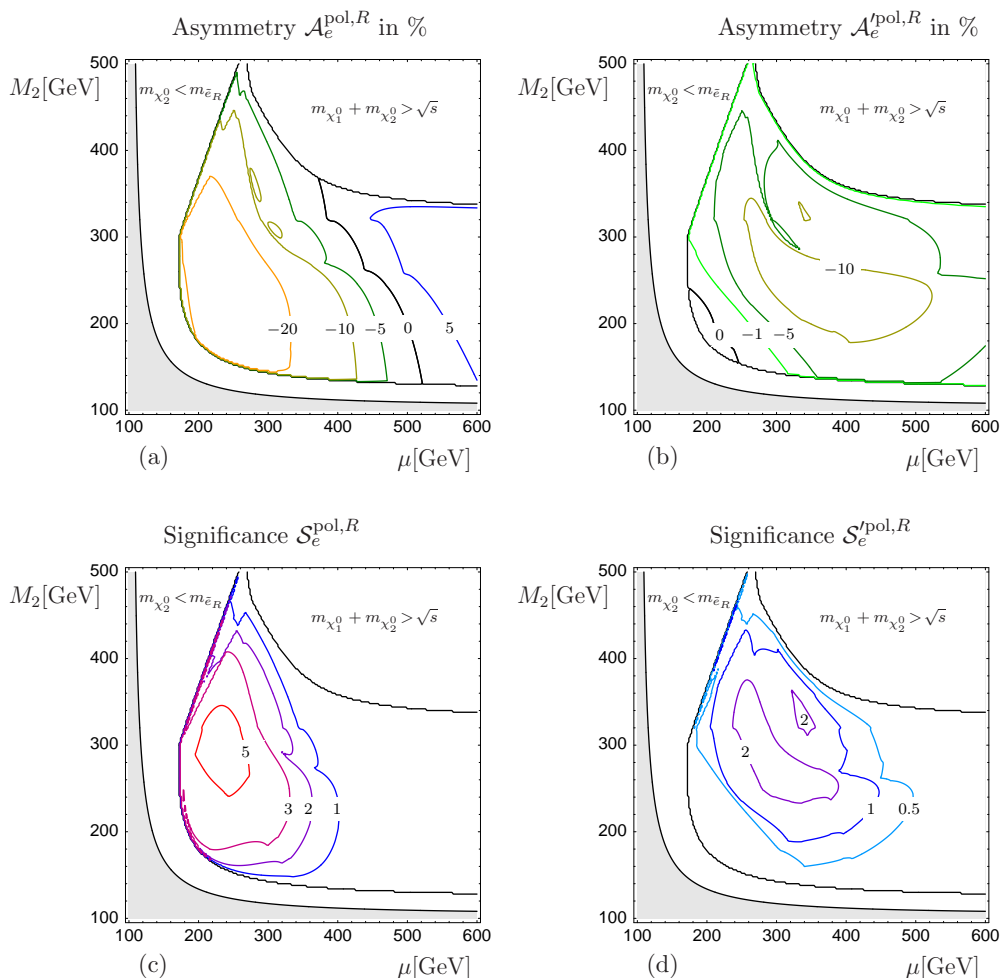


Figure 9: Neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ and decay $\tilde{\chi}_2^0 \rightarrow e\tilde{e}_R$ at $\sqrt{s} = (M_{H_2} + M_{H_3})/2$ with longitudinally polarized beams $\mathcal{P}_- = \mathcal{P}_+ = \pm 0.3$. Contour lines in the μ - M_2 plane for (a) the CP-even polarization asymmetry $\mathcal{A}_e^{\text{pol},R}$, eq. (3.8), (b) the CP-odd polarization asymmetry $\mathcal{A}_e^{\prime\text{pol},R}$, eq. (3.9), and (c) the significance $\mathcal{S}_e^{\text{pol},R}$, eq. (H.3), and (d) the significance $\mathcal{S}_e^{\prime\text{pol},R}$, eq. (H.4), with the effective luminosity $\mathcal{L}_{\text{eff}} = 1 \text{ fb}^{-1}$. The SUSY parameters are given in table 1. The neutralino production cross section, $\sigma(\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ and branching ratio $\text{BR}(\tilde{\chi}_2^0 \rightarrow e^+\tilde{e}_R^-)$ are shown in figure 8. The shaded area is excluded by requiring $m_{\chi_{1,2}^\pm} > 103 \text{ GeV}$.

and charginos in the lower left corner of figure 9(a), where the larger Higgs widths counter the effect of the larger Higgs mass difference. On the contrary, in that region the CP-odd asymmetry $\mathcal{A}_e^{\prime\text{pol},R}$ is reduced due to smaller absorptive phases. Finally, at threshold the longitudinal polarization of the neutralino Σ_{res}^3 (2.15) vanishes, and thus also both decay asymmetries, as follows from eqs. (2.19) and (2.20).

The significance of the CP-odd polarization asymmetry, defined in appendix H, reaches $\mathcal{S}_e^{\prime\text{pol},R} \approx 2$, see figure 9(c), and thus the measurement of this asymmetry will be challenging. Nonetheless, only the asymmetry $\mathcal{A}_e^{\prime\text{pol},n}$ allows to measure the CP-odd contribution b_0 to the longitudinal neutralino polarization Σ_{res}^3 (2.15). However, taking the other leptonic

neutralino decay modes into account, in particular $\tilde{\chi}_2^0 \rightarrow \tau\tilde{\tau}_1$, to analyze $\mathcal{A}_\tau^{\prime\text{pol},1}$, results in larger significances.³

4.3.1 Neutralino decay into a stau-tau pair

The CP-even and CP-odd neutralino polarization asymmetries can also be measured for the neutralino decay into a tau, $\tilde{\chi}_2^0 \rightarrow \tau\tilde{\tau}_1$. Due to mixing in the stau sector, the asymmetries for the decay into a tau are generally smaller than those for the decay into an electron (or muon), $\mathcal{A}_\tau^{\text{pol},1} = \eta_\tau^1 \mathcal{A}_e^{\text{pol},1}$ and $\mathcal{A}_\tau^{\prime\text{pol},1} = \eta_\tau^1 \mathcal{A}_e^{\prime\text{pol},R}$, with $|\eta_\tau^1| \leq 1$, see eq. (3.11). In figure 10(b), we show the contour lines of the reduction factor η_τ^1 (2.32). In the following, we discuss the CP-even asymmetry $\mathcal{A}_\tau^{\text{pol},1}$ only. A similar discussion holds however qualitatively also for the CP-odd asymmetry $\mathcal{A}_\tau^{\prime\text{pol},1}$. Note that a measurement of the τ asymmetries is more involved due to τ -reconstruction efficiencies, which we however neglect in the following for simplicity.

The CP-even asymmetry $\mathcal{A}_\tau^{\text{pol},1}$ is shown in figure 10(c). The significance for measuring the asymmetry also depends on the cross section for production and decay, and thus on the different leptonic branching ratios of the neutralino. The neutralino decay into a tau dominates for $M_2 \lesssim 200$ GeV, see figure 10(c), whereas the branching ratio into an electron can attain more than $\text{BR}(\tilde{\chi}_2^0 \rightarrow e^+ \tilde{e}_R^-) = 16\%$, for $\mu < M_2$, see the contour line in figure 8(b). We take account of this interplay between the size of the branching ratios and the asymmetries by comparing their statistical significances $\mathcal{S}_e^{\text{pol},R}$ and $\mathcal{S}_\tau^{\text{pol},1}$, which we define in appendix H, eq. (H.3). The significances quantify the feasibility of measuring the asymmetries. Both significances can be as large as 5, however in different regions of the μ - M_2 plane, compare figure 9(c) and figure 10(d), respectively.

5. Summary and conclusions

We have analyzed neutralino production and their leptonic decays at the muon collider with longitudinally polarized beams. We have defined polarization asymmetries to study the interference of the heavy neutral MSSM Higgs bosons with CP violation, radiatively induced by the common phase ϕ_A of the trilinear scalar coupling parameter. For nearly degenerate neutral Higgs bosons, as in the Higgs decoupling limit, the CP violating Higgs mixing can be resonantly enhanced, which allows for large CP violating effects.

For neutralino production, we have defined a CP-odd asymmetry of the cross section for equal positive and negative muon beam polarizations. The CP-odd production asymmetry is sensitive to the CP-phases in the Higgs sector, and also receives large contributions from absorptive phases of the Higgs propagators. In a numerical study, we have obtained large values of the production asymmetry up to 40% for equal beam polarizations of $\mathcal{P} = 0.3$. For neutralinos with mixed gaugino-higgsino character, the production cross section can be as large as 1500 fb. Thus the asymmetry can be measured with high statistical significance.

³The decay asymmetries $\mathcal{A}_e^{(\prime)\text{pol},L}$, $\mathcal{A}_\mu^{(\prime)\text{pol},L}$, and $\mathcal{A}_\tau^{(\prime)\text{pol},2}$ are only accessible for $\mu \gtrsim 500$ GeV and $M_2 \gtrsim 200$ GeV in our scenario. They are not relevant for our discussion, since the corresponding branching ratios of $\tilde{\chi}_2^0$ are only as large as a few percent.

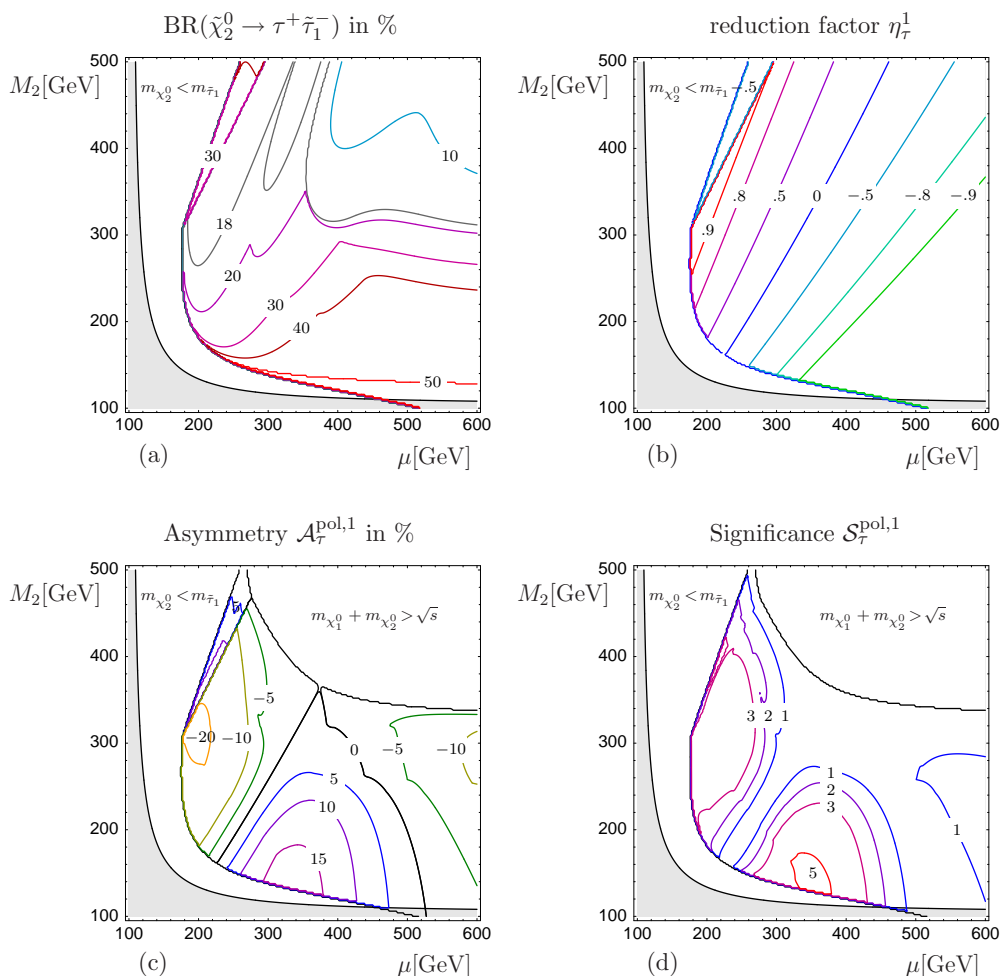


Figure 10: Neutralino production $\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ and decay into a tau $\tilde{\chi}_2^0 \rightarrow \tau\tilde{\tau}_1$, at $\sqrt{s} = (M_{H_2} + M_{H_3})/2$ with longitudinally polarized beams $\mathcal{P}_- = \mathcal{P}_+ = \pm 0.3$. Contour lines in the μ - M_2 plane for (a) the branching ratio $\text{BR}(\tilde{\chi}_2^0 \rightarrow \tau^+\tilde{\tau}_1^-)$, (b) the factor η_τ^1 (2.32), (c) the CP-even polarization asymmetry $\mathcal{A}_\tau^{\text{pol},1}$, eq. (3.8), and (d) the significance $\mathcal{S}_\tau^{\text{pol},1}$, eq. (H.3), with the effective luminosity $\mathcal{L}_{\text{eff}} = 1 \text{ fb}^{-1}$, for the SUSY parameters as given in table 1. The shaded area is excluded by requiring $m_{\tilde{\chi}_1^\pm} > 103 \text{ GeV}$. The cross section $\sigma(\mu^+\mu^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0)$ is shown in figure 8.

We have shown that also the neutralino polarization depends sensitively on the Higgs interference. For the subsequent leptonic decays of the neutralino, we have analyzed two asymmetries of the energy distributions of the final leptons e , μ and τ . The decay asymmetries probe the CP-odd and the CP-even contributions to the longitudinal neutralino polarization, respectively. The decay asymmetries are complementary to the production asymmetry, since they strongly depend on spin-correlations. The CP-even asymmetry is due to a correlation between the longitudinal polarizations of the initial muons and the final neutralinos. Being CP-even, the asymmetry reaches 25% for vanishing CP-phases, and is reduced in the presence of CP-phases. The CP-odd asymmetry is due to the spin

correlations in the neutralino production and decay process. Similarly to the CP-odd asymmetry from the production, this decay asymmetry is approximately maximal if the scalar-pseudoscalar Higgs mixing is resonantly enhanced, which appears naturally in the Higgs decoupling limit. The decay asymmetries yield additional information on the CP nature of the Higgs resonances, and complement the production asymmetry. The asymmetries thus allow a systematic study of the interference and mixing effects of CP violating neutral Higgs sector at the muon collider.

Acknowledgments

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A. Note on the effective Higgs couplings

In section 2.1, we have defined the effective couplings of the Higgs bosons to muons and neutralinos. They are obtained by rotating the tree level Higgs couplings by the matrix C . This matrix includes the leading radiative self-energy corrections, and is defined by diagonalizing the Higgs propagator matrix and the Higgs mass matrix

$$\Delta_D(p^2) = C\Delta(p^2)C^{-1}, \quad \mathbf{M}_D(p^2) = C\mathbf{M}(p^2)C^{-1}, \quad (\text{A.1})$$

respectively, at fixed momentum squared $p^2 = M_{H_2}^2$. The weak momentum dependence of the mass matrix \mathbf{M} is neglected, as in the Weisskopf-Wigner approximation [39]. Due to the absorptive parts of the transition amplitude, the matrix C is in general non-unitary. Thus the transformed basis of the approximate⁴ Higgs boson fields $\{H_1, H_2, H_3\}$, is non-orthonormal [10]. As a consequence, there exists a dual basis $\{\tilde{H}_1, \tilde{H}_2, \tilde{H}_3\}$ obtained by the matrix $\tilde{C} = C^{-1T}$. The corresponding states satisfy the orthogonality relations $\langle H_k | \tilde{H}_l \rangle = \delta_{kl}$. This leads to different transformations of the tree-level Higgs couplings to initial and final state fermions with C and \tilde{C} , relations (2.5) and (2.6), respectively. This follows, since the amplitude (2.10) for neutralino production can also be written in the general form

$$\begin{aligned} T^P &= \Gamma^{(\chi)} \Delta \Gamma^{(\mu)} \\ &= \Gamma^{(\chi)} C^{-1} C \Delta C^{-1} C \Gamma^{(\mu)} \\ &= \Gamma^{(\chi)} C^{-1} \Delta_D C \Gamma^{(\mu)} = \Gamma_{\text{eff}}^{(\chi)} \Delta_D \Gamma_{\text{eff}}^{(\mu)}, \end{aligned} \quad (\text{A.2})$$

where $\Gamma^{(\chi)}$ and $\Gamma^{(\mu)}$ are the one-particle irreducible Higgs vertices to muons and neutralinos, respectively. Eq. (A.2) defines the effective one-particle irreducible Higgs vertices for initial and final fermion states,

$$\Gamma_{l \text{ eff}}^{(\mu)} = C_{lj} \Gamma_j^{(\mu)}, \quad (\text{A.3})$$

$$\Gamma_{k \text{ eff}}^{(\chi)} = \tilde{C}_{ki} \Gamma_i^{(\chi)}, \quad (\text{A.4})$$

⁴We call the Higgs basis $\{H_1, H_2, H_3\}$ approximate, since it corresponds to $\mathbf{M}_D(p^2)$ with fixed momentum squared $p^2 = M_{H_2}^2$, assuming a weak momentum dependence in the resonance region.

which transform with C and \tilde{C} , respectively. If the phases of the Higgs boson states are chosen appropriately, the matrix C can be made complex orthogonal [52], which implies $\tilde{C} = C$.

B. Correlation between initial and final longitudinal polarizations

In this appendix, we analyze the correlation between initial and final longitudinal polarizations in neutralino pair production in $\mu^+\mu^-$ annihilation via Higgs boson exchange in the simplified case of a CP conserving Higgs sector.

Helicity and CP eigenstates

Out of four possible spin/helicity states of a fermion pair only those with $J_z = 0$ in the center-of-mass system (CMS) interact with the Higgs bosons. Here J denotes the total angular momentum and \hat{z} the direction of the momenta of the fermions. Since in the CMS the orbital angular momentum of the fermions is orthogonal to their momenta, $L_z = 0$, their total spin S satisfies $S_z = J_z - L_z = 0$. The fermion interacting states are thus $|LL\rangle_{ff'}$ and $|RR\rangle_{ff'}$, where L and R denote the helicities of the fermions $ff' = \mu^+\mu^-$, $\tilde{\chi}_i^0\tilde{\chi}_j^0$. These states are linear combinations of states $|S, S_z\rangle_{ff'}$ with spin $S = 0$ and $S = 1$,

$$|LL\rangle_{ff'} = \frac{1}{\sqrt{2}}(|1, 0\rangle - |0, 0\rangle)_{ff'}, \quad (\text{B.1})$$

$$|RR\rangle_{ff'} = \frac{1}{\sqrt{2}}(|1, 0\rangle + |0, 0\rangle)_{ff'}. \quad (\text{B.2})$$

For a fermion-antifermion system their spin S is related to their CP quantum number by $\text{CP} = \eta_{ff'}(-1)^{S+1}$, with $\eta_{f\bar{f}} = 1$ for a Dirac fermion-antifermion pair and $\eta_{\tilde{\chi}_i^0\tilde{\chi}_j^0} \equiv \eta_{ij} = e^{2i\sigma_{ij}}$ for a pair of neutralinos. The relative CP phase factor η_{ij} is real in our analysis since the neutralino sector is CP-conserving, with $\sigma_{ij} = 0, \pi/2$. The CP-even and CP-odd muon states

$$|\text{CP}+\rangle_{\mu^+\mu^-} = |1, 0\rangle_{\mu^+\mu^-}, \quad (\text{B.3})$$

$$|\text{CP}-\rangle_{\mu^+\mu^-} = i|0, 0\rangle_{\mu^+\mu^-}, \quad (\text{B.4})$$

and neutralino states

$$|\text{CP}+\rangle_{\tilde{\chi}_i^0\tilde{\chi}_j^0} = (\cos\sigma_{ij}|1, 0\rangle + i\sin\sigma_{ij}|0, 0\rangle)_{\tilde{\chi}_i^0\tilde{\chi}_j^0}, \quad (\text{B.5})$$

$$|\text{CP}-\rangle_{\tilde{\chi}_i^0\tilde{\chi}_j^0} = (-\sin\sigma_{ij}|1, 0\rangle + i\cos\sigma_{ij}|0, 0\rangle)_{\tilde{\chi}_i^0\tilde{\chi}_j^0}, \quad (\text{B.6})$$

can be expressed as a linear combination of helicity states inverting eqs. (B.1) and (B.2). Analogously, the helicity states are linear combinations of the CP-even and CP-odd states. For the muon-antimuon pairs we obtain

$$|LL\rangle_{\mu^+\mu^-} = \frac{1}{\sqrt{2}}(|\text{CP}+\rangle + i|\text{CP}-\rangle)_{\mu^+\mu^-}, \quad (\text{B.7})$$

$$|RR\rangle_{\mu^+\mu^-} = \frac{1}{\sqrt{2}}(|\text{CP}+\rangle - i|\text{CP}-\rangle)_{\mu^+\mu^-}. \quad (\text{B.8})$$

Transition amplitudes

Assuming a CP conserving Higgs sector implies that, in our Higgs mediated neutralino

production process, a CP-even $\mu^+\mu^-$ state $|\text{CP}+\rangle_{\mu^+\mu^-}$ can only produce CP-even Higgs bosons, which in turn decay into the CP-even neutralino state (B.5) with a real amplitude α . Analogously, a CP-odd $\mu^+\mu^-$ state leads to a CP-odd neutralino state (B.6) with an amplitude β .

An initial state with right handed polarized muon and antimuons (B.8), will interact to produce the neutralino state

$$\begin{aligned} |RR\rangle_{\mu^+\mu^-} &\rightarrow \sqrt{2}\mathcal{N}'_R[\alpha|\text{CP}+\rangle - i\beta|\text{CP}-\rangle]_{\chi_i^0\chi_j^0}, \\ &= \mathcal{N}'_R[e^{i\sigma_{ij}}(\alpha + \beta)|RR\rangle + e^{-i\sigma_{ij}}(\alpha - \beta)|LL\rangle]_{\chi_i^0\chi_j^0}, \end{aligned} \tag{B.9}$$

where \mathcal{N}'_R is a normalization factor. Here we have used the explicit form of the neutralino states (B.5) and (B.6) and have inverted eqs. (B.1) and (B.2).

Similarly,

$$|LL\rangle_{\mu^+\mu^-} \rightarrow \mathcal{N}'_L[e^{i\sigma_{ij}}(\alpha - \beta)|RR\rangle + e^{-i\sigma_{ij}}(\alpha + \beta)|LL\rangle]_{\chi_i^0\chi_j^0}. \tag{B.10}$$

From eqs. (B.9) and (B.10) follows that if either the CP-even or CP-odd amplitudes, α or β , respectively, vanish, then so does the neutralino polarization, since in this case the absolute value of the coefficients on the r.h.s. of eqs. (B.9) and (B.10) are equal. Note that the neutralino polarization depends on the relative signs of the CP-even and CP-odd amplitudes since it arises from their interference.

From eqs. (B.9) and (B.10) also follows that if the muon beams are not longitudinally polarized, which implies that the initial state has equal proportions of left and right handed $\mu^+\mu^-$ states (B.7) and (B.8), the final neutralinos will also be unpolarized.

Concluding, in the CP conserving Higgs sector the neutralino polarization is sensitive to the relative sign of the transition amplitudes, and thus to the product of couplings, and can only be non vanishing if the muon beams are longitudinally polarized. This implies that the correlation between initial and final state longitudinal polarizations depends on the interference of transition amplitudes mediated by Higgs bosons of different CP parities.

In the more general CP violating Higgs sector studied in this paper, the abovementioned correlation between initial and final polarizations leads to the CP-even asymmetry $\mathcal{A}_\ell^{\text{pol},n}$ (3.8). In addition, CP-odd (and CPT-odd) effects lead to the CP-odd asymmetry $\mathcal{A}_\ell^{\text{pol},n}$ (3.9). However, since this asymmetry is not due to a correlation between initial and final state polarizations, it can be non-zero even for vanishing beam polarizations.

C. Lagrangians for non-resonant neutralino production and leptonic decay

The non-resonant neutralino production (2.1) proceeds via Z^0 boson exchange in the s -channel, and smuon $\tilde{\mu}_{L,R}$ exchange in the t - and u -channels, see the Feynman diagrams in figure 2. The interaction Lagrangians for neutralino production and those for its leptonic

decay $\tilde{\chi}_1^0 \rightarrow \ell \tilde{\ell}_{L,R}$, with $\ell = e, \mu$ are [53, 41]

$$\mathcal{L}_{Z^0 \tilde{\chi}_i^0 \tilde{\chi}_j^0} = \frac{1}{2} \frac{g}{\cos \theta_W} Z_\nu^0 \tilde{\chi}_i^0 \gamma^\nu [O_{ij}^{\prime\prime L} P_L + O_{ij}^{\prime\prime R} P_R] \tilde{\chi}_j^0, \quad i, j = 1, \dots, 4, \quad (\text{C.1})$$

$$\mathcal{L}_{Z^0 \mu \bar{\mu}} = -\frac{g}{\cos \theta_W} Z_\nu^0 \bar{\mu} \gamma^\nu [L_\mu P_L + R_\mu P_R] \mu, \quad (\text{C.2})$$

$$\mathcal{L}_{\tilde{\ell} \tilde{\chi}_j^0} = g f_{\ell j}^L \bar{\ell} P_R \tilde{\chi}_j^0 \tilde{\ell}_L + g f_{\ell j}^R \bar{\ell} P_L \tilde{\chi}_j^0 \tilde{\ell}_R + \text{h.c.} \quad (\text{C.3})$$

with $P_{L,R} = (1 \mp \gamma_5)/2$. In the photino, zino, higgsino basis $(\tilde{\gamma}, \tilde{Z}, \tilde{H}_a^0, \tilde{H}_b^0)$, the couplings are [53, 41]

$$O_{ij}^{\prime\prime L} = -\frac{1}{2} [(N_{i3} N_{j3}^* - N_{i4} N_{j4}^*) \cos 2\beta + (N_{i3} N_{j4}^* + N_{i4} N_{j3}^*) \sin 2\beta], \quad (\text{C.4})$$

$$O_{ij}^{\prime\prime R} = -O_{ij}^{\prime\prime L*}, \quad (\text{C.5})$$

$$L_\mu = -\frac{1}{2} + \sin^2 \theta_W, \quad R_\mu = \sin^2 \theta_W, \quad (\text{C.6})$$

$$f_{\ell j}^L = -\sqrt{2} \left[\frac{1}{\cos \theta_W} (T_{3\ell} - e_\ell \sin^2 \theta_W) N_{j2} + e_\ell \sin \theta_W N_{j1} \right], \quad (\text{C.7})$$

$$f_{\ell j}^R = -\sqrt{2} e_\ell \sin \theta_W \left[\tan \theta_W N_{j2}^* - N_{j1}^* \right], \quad (\text{C.8})$$

with e_ℓ and $T_{3\ell}$ the electric charge and third component of the weak isospin of the lepton ℓ , the weak mixing angle θ_W , the weak coupling constant $g = e/\sin \theta_W$, $e > 0$, and the ratio $\tan \beta = v_2/v_1$ of the vacuum expectation values of the two neutral Higgs fields. The neutralino couplings to the Z boson, $O_{ij}^{\prime\prime L,R}$, and to the smuons, $f_{\mu i}^{L,R}$, contain the complex mixing elements N_{ij} , which diagonalize the neutralino matrix $N_{i\alpha}^* Y_{\alpha\beta} N_{\beta k}^\dagger = m_{\chi_i} \delta_{ik}$ [1], with neutralino masses $m_{\chi_i} > 0$.

Mixing can safely be neglected for the scalar leptons of the first two generations, $\tilde{\ell} = \tilde{e}, \tilde{\mu}$. For the neutralino decay into staus $\tilde{\chi}_i^0 \rightarrow \tilde{\tau}_n \tau$, $n = 1, 2$, we take stau mixing into account, see appendix G.

D. Density matrix formalism

We use the spin density matrix formalism of [40, 41] for the calculation of the squared amplitudes for neutralino production (2.1) and decay (2.2). The amplitude for neutralino production via resonant Higgs exchange, eq. (D.1), depends on the helicities λ_\pm of the muons μ^\pm and the helicities λ_i, λ_j of the neutralinos $\tilde{\chi}_i^0, \tilde{\chi}_j^0$

$$\begin{aligned} T_{\lambda_i \lambda_j \lambda_+ \lambda_-}^P &= \Delta(H_k) \left[\bar{v}(p_{\mu^+}, \lambda_+) \left(c_L^{H_k \mu \mu} P_L + c_R^{H_k \mu \mu} P_R \right) u(p_{\mu^-}, \lambda_-) \right] \\ &\quad \times \left[\bar{u}(p_{\chi_j^0}, \lambda_j) \left(c_L^{H_k \chi_i \chi_j} P_L + c_R^{H_k \chi_i \chi_j} P_R \right) v(p_{\chi_i^0}, \lambda_i) \right]. \end{aligned} \quad (\text{D.1})$$

We include the longitudinal beam polarizations of the muon-beams, \mathcal{P}_- and \mathcal{P}_+ , with $-1 \leq \mathcal{P}_\pm \leq +1$ in their density matrices

$$\rho_{\lambda_- \lambda'_-}^- = \frac{1}{2} \left(\delta_{\lambda_- \lambda'_-} + \mathcal{P}_- \tau_{\lambda_- \lambda'_-}^3 \right), \quad (\text{D.2})$$

$$\rho_{\lambda_+ \lambda'_+}^+ = \frac{1}{2} \left(\delta_{\lambda_+ \lambda'_+} + \mathcal{P}_+ \tau_{\lambda_+ \lambda'_+}^3 \right), \quad (\text{D.3})$$

where τ^3 is the third Pauli matrix. The unnormalized spin density matrix of $\tilde{\chi}_i^0 \tilde{\chi}_j^0$ production and $\tilde{\chi}_j^0$ decay are given by, respectively,

$$\rho_{\lambda_j \lambda'_j}^P = \sum_{\lambda_i, \lambda_+, \lambda'_+, \lambda_-, \lambda'_-} \rho_{\lambda_+ \lambda'_+}^+ \rho_{\lambda_- \lambda'_-}^- T_{\lambda_i \lambda_j \lambda_+ \lambda_-}^P T_{\lambda_i \lambda'_j \lambda'_+ \lambda'_-}^{P*}, \quad (\text{D.4})$$

$$\rho_{\lambda'_j \lambda_j}^D = T_{\lambda'_j}^{D*} T_{\lambda_j}^D. \quad (\text{D.5})$$

The amplitude squared for production and decay is then

$$|T|^2 = |\Delta(\tilde{\chi}_j^0)|^2 \sum_{\lambda_j \lambda'_j} \rho_{\lambda_j \lambda'_j}^P \rho_{\lambda'_j \lambda_j}^D, \quad (\text{D.6})$$

with the neutralino propagator

$$\Delta(\tilde{\chi}_j^0) = \frac{i}{p_{\chi_j^0}^2 - m_{\chi_j^0}^2 + im_{\chi_j^0} \Gamma_{\chi_j^0}}. \quad (\text{D.7})$$

The spin density matrices (D.4) and (D.5) can be expanded in terms of the Pauli matrices τ^a

$$\rho_{\lambda_j \lambda'_j}^P = \delta_{\lambda_j \lambda'_j} P + \sum_{a=1}^3 \tau_{\lambda_j \lambda'_j}^a \Sigma_P^a, \quad (\text{D.8})$$

$$\rho_{\lambda'_j \lambda_j}^D = \delta_{\lambda'_j \lambda_j} D + \sum_{a=1}^3 \tau_{\lambda'_j \lambda_j}^a \Sigma_D^a, \quad (\text{D.9})$$

where we have defined a set of neutralino spin vectors $s_{\chi_j^0}^a$. In the center-of-mass system, they are

$$s_{\chi_j^0}^{1,\mu} = (0; 1, 0, 0), \quad s_{\chi_j^0}^{2,\mu} = (0; 0, 1, 0), \quad s_{\chi_j^0}^{3,\mu} = \frac{1}{m_{\chi_j^0}} \left(|\vec{p}_{\chi_j^0}|; 0, 0, E_{\chi_j^0} \right). \quad (\text{D.10})$$

We have chosen a coordinate frame such that the momentum of the neutralino $\tilde{\chi}_j^0$ is given by

$$p_{\chi_j^0}^\mu = \left(E_{\chi_j^0}; 0, 0, |\vec{p}_{\chi_j^0}| \right), \quad (\text{D.11})$$

with

$$E_{\chi_j^0} = \frac{s + m_{\chi_j^0}^2 - m_{\chi_i^0}^2}{2\sqrt{s}}, \quad |\vec{p}_{\chi_j^0}| = \frac{\sqrt{\lambda_{ij}}}{2\sqrt{s}}, \quad (\text{D.12})$$

and the triangle function

$$\lambda_{ij} = \lambda(s, m_{\chi_i^0}^2, m_{\chi_j^0}^2), \quad (\text{D.13})$$

with $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$.

Inserting the density matrices (D.8) and (D.9) into (D.6), gives then the amplitude squared in the form of eq. (2.12).

E. Continuum amplitudes and contributions

The amplitudes for non-resonant Z and $\tilde{\mu}_{L,R}$ exchange are

$$T_{\lambda_i \lambda_j \lambda_+ \lambda_-}^P(s, Z) = \frac{g^2}{\cos^2 \theta_W} \Delta^s(Z) [\bar{v}(p_{\mu^+}, \lambda_+) \gamma^\mu (L_\mu P_L + R_\mu P_R) u(p_{\mu^-}, \lambda_-)] \\ \times \left[\bar{u}(p_{\chi_j^0}, \lambda_j) \gamma_\mu (O_{ji}^{\prime\prime L} P_L + O_{ji}^{\prime\prime R} P_R) v(p_{\chi_i^0}, \lambda_i) \right], \quad (\text{E.1})$$

$$T_{\lambda_i \lambda_j \lambda_+ \lambda_-}^P(t, \tilde{\mu}_L) = -g^2 f_{\mu i}^L f_{\mu j}^{L*} \Delta^t(\tilde{\mu}_L) [\bar{v}(p_{\mu^+}, \lambda_+) P_R v(p_{\chi_i^0}, \lambda_i)] \\ \times \left[\bar{u}(p_{\chi_j^0}, \lambda_j) P_L u(p_{\mu^-}, \lambda_-) \right], \quad (\text{E.2})$$

$$T_{\lambda_i \lambda_j \lambda_+ \lambda_-}^P(t, \tilde{\mu}_R) = -g^2 f_{\mu i}^R f_{\mu j}^{R*} \Delta^t(\tilde{\mu}_R) [\bar{v}(p_{\mu^+}, \lambda_+) P_L v(p_{\chi_i^0}, \lambda_i)] \\ \times \left[\bar{u}(p_{\chi_j^0}, \lambda_j) P_R u(p_{\mu^-}, \lambda_-) \right], \quad (\text{E.3})$$

$$T_{\lambda_i \lambda_j \lambda_+ \lambda_-}^P(u, \tilde{\mu}_L) = g^2 f_{\mu i}^{L*} f_{\mu j}^L \Delta^u(\tilde{\mu}_L) [\bar{v}(p_{\mu^+}, \lambda_+) P_R v(p_{\chi_j^0}, \lambda_j)] \\ \times \left[\bar{u}(p_{\chi_i^0}, \lambda_i) P_L u(p_{\mu^-}, \lambda_-) \right], \quad (\text{E.4})$$

$$T_{\lambda_i \lambda_j \lambda_+ \lambda_-}^P(u, \tilde{\mu}_R) = g^2 f_{\mu i}^{R*} f_{\mu j}^R \Delta^u(\tilde{\mu}_R) [\bar{v}(p_{\mu^+}, \lambda_+) P_L v(p_{\chi_j^0}, \lambda_j)] \\ \times \left[\bar{u}(p_{\chi_i^0}, \lambda_i) P_R u(p_{\mu^-}, \lambda_-) \right], \quad (\text{E.5})$$

with the propagators

$$\Delta^s(Z) = \frac{i}{s - m_Z^2}, \quad \Delta^t(\tilde{\mu}_{R,L}) = \frac{i}{t - m_{\tilde{\mu}_{R,L}}^2}, \quad \Delta^u(\tilde{\mu}_{R,L}) = \frac{i}{u - m_{\tilde{\mu}_{R,L}}^2}, \quad (\text{E.6})$$

and $t = (p_{\mu^-} - p_{\chi_j^0})^2$ and $u = (p_{\mu^-} - p_{\chi_i^0})^2$. We neglect the Z -width in the propagator $\Delta^s(Z)$ for energies beyond the resonance. The Feynman diagrams are shown in figure 2. For e^+e^- collisions, the amplitudes are given in [41].

The continuum contributions P_{cont} are those from the non-resonant Z and $\tilde{\mu}_{L,R}$ exchange channels. The coefficient P_{cont} is independent of the neutralino polarization. It can be decomposed into contributions from the different continuum channels

$$P_{\text{cont}} = P(ZZ) + P(Z\tilde{\mu}_R) + P(\tilde{\mu}_R\tilde{\mu}_R) + P(Z\tilde{\mu}_L) + P(\tilde{\mu}_L\tilde{\mu}_L), \quad (\text{E.7})$$

with

$$P(ZZ) = 4 \frac{g^4}{\cos^4 \theta_W} |\Delta^s(Z)|^2 (R_\mu^2 c_R + L_\mu^2 c_L) E_b^2 \\ \times \left\{ |O_{ij}^{\prime\prime R}|^2 (E_{\chi_i^0} E_{\chi_j^0} + q^2 \cos^2 \theta) \right. \\ \left. - [(Re O_{ij}^{\prime\prime R})^2 - (Im O_{ij}^{\prime\prime R})^2] m_{\chi_i^0} m_{\chi_j^0} \right\}, \quad (\text{E.8})$$

$$P(Z\tilde{\mu}_R) = \frac{2g^4}{\cos^2 \theta_W} R_\mu c_R E_b^2 Re \left\{ \Delta^s(Z) \right. \\ \times \left[-(\Delta^{t*}(\tilde{\mu}_R) f_{\mu i}^{R*} f_{\mu j}^R O_{ij}^{\prime\prime R*} + \Delta^{u*}(\tilde{\mu}_R) f_{\mu i}^R f_{\mu j}^{R*} O_{ij}^{\prime\prime R}) m_{\chi_i^0} m_{\chi_j^0} \right. \\ \left. + (\Delta^{t*}(\tilde{\mu}_R) f_{\mu i}^{R*} f_{\mu j}^R O_{ij}^{\prime\prime R} + \Delta^{u*}(\tilde{\mu}_R) f_{\mu i}^R f_{\mu j}^{R*} O_{ij}^{\prime\prime R*}) (E_{\chi_i^0} E_{\chi_j^0} + q^2 \cos^2 \theta) \right. \\ \left. - (\Delta^{t*}(\tilde{\mu}_R) f_{\mu i}^{R*} f_{\mu j}^R O_{ij}^{\prime\prime R} - \Delta^{u*}(\tilde{\mu}_R) f_{\mu i}^R f_{\mu j}^{R*} O_{ij}^{\prime\prime R*}) 2E_b q \cos \theta \right] \right\}, \quad (\text{E.9})$$

$$\begin{aligned}
 P(\tilde{\mu}_R \tilde{\mu}_R) = & \frac{g^4}{2} c_R E_b^2 \left\{ |f_{\mu i}^R|^2 |f_{\mu j}^R|^2 \times \right. \\
 & \left[(|\Delta^t(\tilde{\mu}_R)|^2 + |\Delta^u(\tilde{\mu}_R)|^2) (E_{\chi_i^0} E_{\chi_j^0} + q^2 \cos^2 \theta) \right. \\
 & \left. - (|\Delta^t(\tilde{\mu}_R)|^2 - |\Delta^u(\tilde{\mu}_R)|^2) 2E_b q \cos \theta \right] \\
 & \left. - R e \{ (f_{\mu i}^{R*})^2 (f_{\mu j}^R)^2 \Delta^u(\tilde{\mu}_R) \Delta^{t*}(\tilde{\mu}_R) \} 2m_{\chi_i^0} m_{\chi_j^0} \right\}. \quad (\text{E.10})
 \end{aligned}$$

To obtain the quantities $P(Z\tilde{\mu}_L), P(\tilde{\mu}_L\tilde{\mu}_L)$ one has to exchange in (E.9) and (E.10)

$$\begin{aligned}
 \Delta^t(\tilde{\mu}_R) & \rightarrow \Delta^t(\tilde{\mu}_L), & \Delta^u(\tilde{\mu}_R) & \rightarrow \Delta^u(\tilde{\mu}_L), & c_R & \rightarrow c_L \\
 R_\mu & \rightarrow L_\mu, & O_{ij}^{\prime\prime R} & \rightarrow O_{ij}^{\prime\prime L}, & f_{\mu i}^R & \rightarrow f_{\mu i}^L, & f_{\mu j}^R & \rightarrow f_{\mu j}^L. \quad (\text{E.11})
 \end{aligned}$$

The longitudinal beam polarizations are included in the weighting factors

$$c_L = (1 - \mathcal{P}_-)(1 + \mathcal{P}_+), \quad c_R = (1 + \mathcal{P}_-)(1 - \mathcal{P}_+). \quad (\text{E.12})$$

For e^+e^- collisions, the P terms are also given in [41], however they differ by a factor of 2 in our notation (2.12). The continuum contributions Σ_{cont}^a (2.13) to the neutralino polarization can also be found in [41], also differing by a factor of 2. However, due to the Majorana character of the neutralinos, the continuum contributions Σ_{cont}^a are forward-backward antisymmetric, and vanish if integrated over the neutralino production angle [43], see eq. (2.33).

F. Neutralino decay into leptons

The expansion coefficients of the decay matrix (D.9) for the neutralino decay into right sleptons $\tilde{\chi}_j^0 \rightarrow \ell^+ \tilde{\ell}_R^-$, with $\ell = e, \mu$, are

$$D = \frac{g^2}{2} |f_{\ell i}^R|^2 \left(m_{\chi_i^0}^2 - m_{\tilde{\ell}}^2 \right), \quad (\text{F.1})$$

$$\Sigma_D^a = +g^2 |f_{\ell i}^R|^2 m_{\chi_j^0} \left(s_{\chi_j^0}^a \cdot p_{\ell_1} \right). \quad (\text{F.2})$$

For the decay into the left sleptons $\tilde{\chi}_j^0 \rightarrow \ell^+ \tilde{\ell}_L^-$, $\ell = e, \mu$, the coefficients are

$$D = \frac{g^2}{2} |f_{\ell j}^L|^2 \left(m_{\chi_j^0}^2 - m_{\tilde{\ell}}^2 \right), \quad (\text{F.3})$$

$$\Sigma_D^a = -g^2 |f_{\ell i}^L|^2 m_{\chi_j^0} \left(s_{\chi_j^0}^a \cdot p_{\ell_1} \right). \quad (\text{F.4})$$

For the decay into the stau $\tilde{\chi}_j^0 \rightarrow \tau^+ \tilde{\tau}_k^-$, $k = 1, 2$, one obtains

$$D = \frac{g^2}{2} (|a_{kj}^{\tilde{\tau}}|^2 + |b_{kj}^{\tilde{\tau}}|^2) \left(m_{\chi_j^0}^2 - m_{\tilde{\tau}_k}^2 \right), \quad (\text{F.5})$$

$$\Sigma_D^a = -g^2 (|a_{kj}^{\tilde{\tau}}|^2 - |b_{kj}^{\tilde{\tau}}|^2) m_{\chi_j^0} \left(s_{\chi_j^0}^a \cdot p_{\ell_1} \right). \quad (\text{F.6})$$

The coefficients Σ_D^a for the charge conjugated processes, $\tilde{\chi}_j^0 \rightarrow \ell^- \tilde{\ell}^+$, is obtained by inverting the signs of (F.2), (F.4) and (F.6).

With these definitions we can rewrite the factor Σ_D^3 , that multiplies the longitudinal neutralino polarization Σ_P^3 in (2.12),

$$\Sigma_D^3 = \eta_{\ell\pm} \frac{D}{\Delta_\ell} (E_\ell - \bar{E}_\ell), \quad (\text{F.7})$$

where we have used

$$m_{\chi_j^0}(s_{\chi_j^0}^3 \cdot p_\ell) = -\frac{m_{\chi_j^0}^2}{|\vec{p}_{\chi_j^0}|} (E_\ell - \bar{E}_\ell). \quad (\text{F.8})$$

In order to reduce the free MSSM parameters, we parametrize the slepton masses with their approximate renormalization group equations (RGE) [54]

$$m_{\tilde{\ell}_R}^2 = m_0^2 + m_\ell^2 + 0.23M_2^2 - m_Z^2 \cos 2\beta \sin^2 \theta_W, \quad (\text{F.9})$$

$$m_{\tilde{\ell}_L}^2 = m_0^2 + m_\ell^2 + 0.79M_2^2 + m_Z^2 \cos 2\beta \left(-\frac{1}{2} + \sin^2 \theta_W \right), \quad (\text{F.10})$$

$$m_{\tilde{\nu}_\ell}^2 = m_0^2 + m_\ell^2 + 0.79M_2^2 + \frac{1}{2}m_Z^2 \cos 2\beta, \quad (\text{F.11})$$

with m_0 the common scalar mass parameter the GUT scale.

G. Stau-neutralino couplings

For the neutralino decay into staus $\tilde{\chi}_i^0 \rightarrow \tilde{\tau}_k \tau$, we take stau mixing into account and write for the Lagrangian [55]:

$$\mathcal{L}_{\tau\tilde{\tau}\chi_i} = g\tilde{\tau}_k\bar{\tau} (a_{ki}^{\tilde{\tau}} P_R + b_{ki}^{\tilde{\tau}} P_L) \chi_i^0 + \text{h.c.}, \quad k = 1, 2; \quad i = 1, \dots, 4, \quad (\text{G.1})$$

with

$$a_{kj}^{\tilde{\tau}} = (\mathcal{R}_{kn}^{\tilde{\tau}})^* \mathcal{A}_{jn}^\tau, \quad b_{kj}^{\tilde{\tau}} = (\mathcal{R}_{kn}^{\tilde{\tau}})^* \mathcal{B}_{jn}^\tau, \quad (n = L, R), \quad (\text{G.2})$$

$$\mathcal{A}_j^\tau = \begin{pmatrix} f_{\tau j}^L \\ h_{\tau j}^R \end{pmatrix}, \quad \mathcal{B}_j^\tau = \begin{pmatrix} h_{\tau j}^L \\ f_{\tau j}^R \end{pmatrix}, \quad (\text{G.3})$$

with $\mathcal{R}_{kn}^{\tilde{\tau}}$ given in (G.9). The couplings $f_{\tau j}^L$ and $f_{\tau j}^R$ are defined by eqs. (C.7) and (C.8), respectively, and

$$h_{\tau j}^L = (h_{\tau j}^R)^* = -Y_\tau (N_{j3}^* \cos \beta + N_{j4}^* \sin \beta), \quad (\text{G.4})$$

$$Y_\tau = \frac{m_\tau}{\sqrt{2} m_W \cos \beta}, \quad (\text{G.5})$$

with m_W the mass of the W boson, m_τ the mass of the τ -lepton and N the neutralino mixing matrix in the $\tilde{\gamma}, \tilde{Z}, H_1^0, H_2^0$ basis. The masses and couplings of the τ -sleptons follow from the $\tilde{\tau}_L - \tilde{\tau}_R$ mass matrix

$$\mathcal{L}_M^{\tilde{\tau}} = -(\tilde{\tau}_L^*, \tilde{\tau}_R^*) \begin{pmatrix} m_{\tilde{\tau}_L}^2 & e^{-i\varphi_{\tilde{\tau}}} m_\tau |\Lambda_{\tilde{\tau}}| \\ e^{i\varphi_{\tilde{\tau}}} m_\tau |\Lambda_{\tilde{\tau}}| & m_{\tilde{\tau}_R}^2 \end{pmatrix} \begin{pmatrix} \tilde{\tau}_L \\ \tilde{\tau}_R \end{pmatrix}, \quad (\text{G.6})$$

with $m_{\tilde{\tau}_R}^2$ and $m_{\tilde{\tau}_L}^2$ given by eqs. (F.9) and (F.10) replacing $m_{\tilde{\ell}}^2$ by $m_{\tilde{\tau}}^2$, and

$$\Lambda_{\tilde{\tau}} = A_{\tau} - \mu^* \tan \beta, \quad (\text{G.7})$$

$$\varphi_{\tilde{\tau}} = \arg[\Lambda_{\tilde{\tau}}], \quad (\text{G.8})$$

with $A_{\tau} = A$ the (common) trilinear scalar coupling parameter. The $\tilde{\tau}$ mass eigenstates are $(\tilde{\tau}_1, \tilde{\tau}_2) = (\tilde{\tau}_L, \tilde{\tau}_R)\mathcal{R}^{\tilde{\tau}T}$, with

$$\mathcal{R}^{\tilde{\tau}} = \begin{pmatrix} e^{i\varphi_{\tilde{\tau}}} \cos \theta_{\tilde{\tau}} & \sin \theta_{\tilde{\tau}} \\ -\sin \theta_{\tilde{\tau}} & e^{-i\varphi_{\tilde{\tau}}} \cos \theta_{\tilde{\tau}} \end{pmatrix}. \quad (\text{G.9})$$

The mixing angle is

$$\cos \theta_{\tilde{\tau}} = \frac{-m_{\tau}|\Lambda_{\tilde{\tau}}|}{\sqrt{m_{\tau}^2|\Lambda_{\tilde{\tau}}|^2 + (m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_L}^2)^2}}, \quad \sin \theta_{\tilde{\tau}} = \frac{m_{\tilde{\tau}_L}^2 - m_{\tilde{\tau}_1}^2}{\sqrt{m_{\tau}^2|\Lambda_{\tilde{\tau}}|^2 + (m_{\tilde{\tau}_1}^2 - m_{\tilde{\tau}_L}^2)^2}}, \quad (\text{G.10})$$

and the mass eigenvalues are

$$m_{\tilde{\tau}_{1,2}}^2 = \frac{1}{2} \left[(m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2) \mp \sqrt{(m_{\tilde{\tau}_L}^2 - m_{\tilde{\tau}_R}^2)^2 + 4m_{\tau}^2|\Lambda_{\tilde{\tau}}|^2} \right]. \quad (\text{G.11})$$

H. Statistical significances

We define the statistical significance of the asymmetry $\mathcal{A}_{\text{prod}}^{\text{pol}}$ (3.1) by

$$\mathcal{S}_{\text{prod}}^{\text{pol}} = |\mathcal{A}_{\text{prod}}^{\text{pol}}| \sqrt{2\bar{\sigma}_{ij}\mathcal{L}}, \quad (\text{H.1})$$

where \mathcal{L} denotes the integrated luminosity and

$$\bar{\sigma}_{ij} = \frac{|\sigma_{ij}(\mathcal{P}) - \sigma_{ij}(-\mathcal{P})|}{2} \quad (\text{H.2})$$

is the mean value of the neutralino production cross section σ_{ij} , (2.25), for both equal muon beam polarizations \mathcal{P} and $-\mathcal{P}$. The significances for the polarization asymmetries $\mathcal{A}_{\ell}^{\text{pol},n}$ (3.8) and $\mathcal{A}_{\ell'}^{\text{pol},n}$ (3.9) are defined by

$$\mathcal{S}_{\ell}^{\text{pol},n} = |\mathcal{A}_{\ell}^{\text{pol},n}| \sqrt{4\bar{\sigma}_{ij}\text{BR}(\tilde{\chi}_j^0 \rightarrow \ell^+ \tilde{\ell}_n^-) \mathcal{L}_{\text{eff}}}, \quad (\text{H.3})$$

and

$$\mathcal{S}_{\ell'}^{\text{pol},n} = |\mathcal{A}_{\ell'}^{\text{pol},n}| \sqrt{4\bar{\sigma}_{ij}\text{BR}(\tilde{\chi}_j^0 \rightarrow \ell^+ \tilde{\ell}_n^-) \mathcal{L}_{\text{eff}}}, \quad (\text{H.4})$$

respectively, where $\mathcal{L}_{\text{eff}} = \epsilon_{\ell}^n \mathcal{L}$ is the effective integrated luminosity, with ϵ_{ℓ}^n the detection efficiency of the leptons from the decay $\tilde{\chi}_j^0 \rightarrow \ell^{\pm} \tilde{\ell}_n^{\mp}$. There is a factor 4 appearing in the significances, since the asymmetries require two sets of equal beam polarizations \mathcal{P} , as well as two decay modes, $\tilde{\chi}_j^0 \rightarrow \ell^+ \tilde{\ell}_n^-$, and the charge conjugated decay $\tilde{\chi}_j^0 \rightarrow \ell^- \tilde{\ell}_n^+$.

For an ideal detector, a significance of, e.g., $S = 1$ implies that the asymmetries can be measured at the statistical 68% confidence level. In order to predict the absolute values of confidence levels, clearly detailed Monte Carlo analysis including detector and background simulations with particle identification and reconstruction efficiencies would be required, which is however beyond the scope of the present work.

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